AD-A161 285 ON THE ROLE OF DIMENSIONLESS ELASTIC FRACTURE MECHANICS (U) CARNEGIE-MELLON UNIV PITTSBURGH PA DEPT OF MECHANICAL ENGINEERING G B SINCLAIR 03 JUL 85 SM-85-14 1/2

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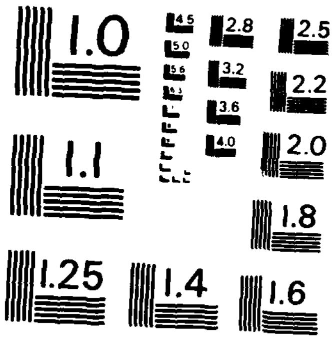

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclasified	1b. RESTRICTIVE MARKINGS None
2a. SECURITY CLASSIFICATION AUTHORITY None	3. DISTRIBUTION/AVAILABILITY OF REPORT Unlimited Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) SM 85-14	5. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 85-0942

6a. NAME OF PERFORMING ORGANIZATION G. B. Sinclair Carnegie-Mellon University	6b. OFFICE SYMBOL (If applicable) N/A	7a. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research
6c. ADDRESS (City, State and ZIP Code) Department of Mechanical Engineering Carnegie-Mellon University Pittsburgh, PA 15213		7b. ADDRESS (City, State and ZIP Code) Bolling Air Force Base Washington, D.C. 20332
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Air Force Office of Scientific Research	8b. OFFICE SYMBOL (If applicable) N/A	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-84-0047

8c. ADDRESS (City, State and ZIP Code)
AFOSR/NA
Building 410
Bolling AFB DC 20332

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2307	B2	

11. TITLE (Include Security Classification) On the role of dimensionless elastic fracture mechanics

12. PERSONAL AUTHOR(S)
G. B. Sinclair

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Final report	FROM 1/15/84 TO 1/14/85	July 3, 1985	116

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	FRACTURE MECHANICS, SIZE EFFECTS, NOTCHED SPECIMENS, STRESS CONCENTRATION, CRACK PROPAGATION

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Dimensionless elastic fracture mechanics (DEFM) - the nondimensionalized counterpart to linear elastic fracture mechanics (LEFM) - predicts size-independent strengths for geometrically similar specimens. This is in contrast to LEFM which has that the stress at fracture reduces as the inverse square root of the in-plane scale factor. It is shown that neither agrees with the data, irrespective of how brittle material response is. Used together with judgement, conceivably a conservative procedure for making strength size predictions is possible. However, both are essentially inadequate, since they lack valid underlying physical reasoning and, even as merely empirically based approaches, are short of sufficient accuracy to be reliable in practice. There is a need, therefore, to critically examine the very foundations of elastic fracture mechanics.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
David A. Glasgow, MAJOR, USAF NA	(202) 767-4987	AFOSR/NA

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE. unclassified

SECURITY CLASSIFICATION OF THIS PAGE

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ON THE ROLE OF DIMENSIONLESS ELASTIC FRACTURE MECHANICS

Final Report on Air Force Office of Scientific

Research Contract No. AFOSR-84-0047

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SM 85-14 July, 1985

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ON THE ROLE OF DIMENSIONLESS ELASTIC FRACTURE MECHANICS

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SUMMARY

Dimensionless elastic fracture mechanics (DEFM) - the nondimensionalized counterpart to linear elastic fracture mechanics (LEFM) - predicts size-independent strengths for geometrically similar specimens. This is in contrast to LEFM which has that the stress at fracture reduces as the inverse square root of the in-plane scale factor. It is shown that neither agrees with the data, irrespective of how brittle material response is. Used together with judgement, conceivably a conservative procedure for making strength size predictions is possible. However, both are essentially inadequate, since they lack valid underlying physical reasoning and, even as merely empirically based approaches, are short of sufficient accuracy to be reliable in practices. There is a need, therefore, to critically examine the very foundations of elastic fracture mechanics.

INTRODUCTION

Being able to predict the strength of geometrically-similar cracked specimens of different sizes or scales is a basic requirement for success for any fracture mechanics technology. The prediction contained in LEFM is that the strength reduces as the inverse square-root of the scale factor in the plane of the crack. To see this, consider the example of a pair of scaled single-edge-cracked specimens shown in Fig. 1. Herein Specimen 1 is a strip of indefinite length yet finite width W, weakened by a crack of length a, and subjected to a remote uniform stress σ_1; while Specimen 2 is also of indefinite length but has width λW, crack length λa, and applied stress σ_2. Thus λ is the in-plane scale factor. For Specimen 1 at fracture, LEFM has

$$K_I = \sigma_1^* \sqrt{\pi a}\, f(a/W) = K_c,$$

where K_I *is the stress intensity factor in Mode I*, *f(a/W)* is a finite width correction factor, K_c is the material fracture toughness, and the asterisk atop σ_1 serves to distinguish it as being the applied stress at fracture. Similarly for Specimen 2, provided it is comprised of the same brittle material, at fracture

$$K_I = \sigma_2^* \sqrt{\pi \lambda a}\, f(\lambda a/\lambda W) = K_c,$$

whence

$$\sigma_1^*/\sigma_2^* = \sqrt{\lambda}. \qquad (1)$$

Given that the underlying continuum mechanics does scale, the size effect evident in (1) is somewhat curious and stems from the choice of the stress intensity factor as the parameter governing brittle fracture. The question examined in this research program is how appropriate is such an absence of scaling.

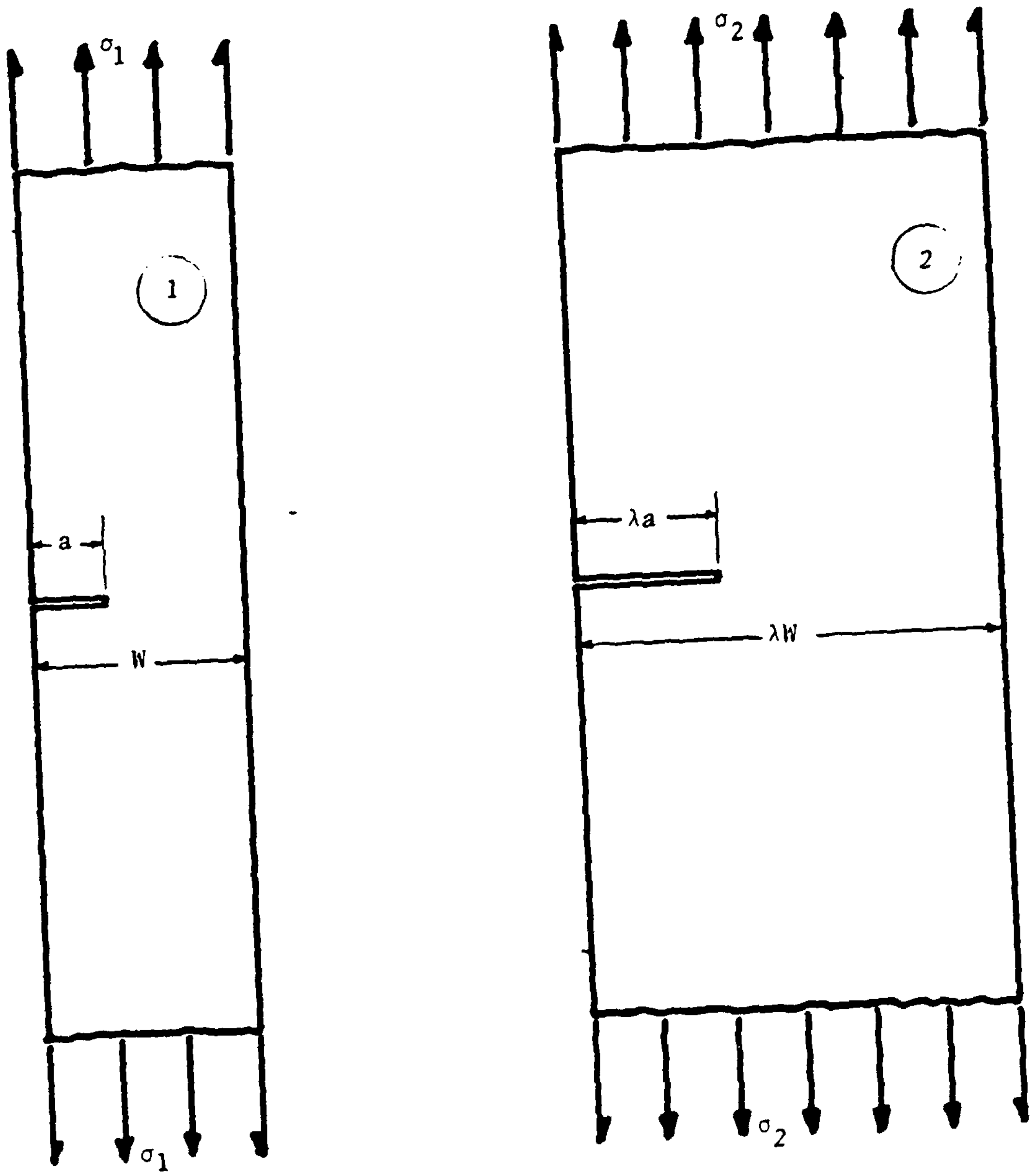

Fig. 1. Scaled pair of single-edge-cracked specimens

The view at the outset of the program was that response should scale unless there are microstructural factors and accompanying lengths involved. To this end, we form dimensionless elastic fracture mechanics simply by replacing K_I by $\bar{K}_I$, the dimensionless stress crack factor. More precisely, in the previous example, we define

$$\bar{K}_I = K_I/\sigma_1\sqrt{W} = \sqrt{\pi a/W}\ f(a/W),$$

for Specimen 1, and assume fracture to be controlled by a material fracture stress, σ_c, so that, at fracture

$$\sigma_1^* \bar{K}_I = \sigma_c.$$

Similarly for Specimen 2,

$$\sigma_2^* \bar{K}_I = \sigma_2^* \sqrt{\pi\lambda a/\lambda W}\ f(\lambda a/\lambda W) = \sigma_c$$

a fracture, whence

$$\sigma_1^*/\sigma_2^* = 1 \qquad (2)$$

That is, the strength scales in the dimensionless version, and it is expected that (2) in fact holds unless there are microstructural size effects. With significant caveats, this view remains the same in the light of the physical evidence examined. The limited role, though, that either LEFM or DEFM can play in predicting strength size effects is now understood more fully. We explain why in what follows.

OVERVIEW OF RESULTS OBTAINED IN RESEARCH PROGRAM

One of the first sets of test performed within this program to see which of (1), (2) best applies was on a model material - Xerox paper embrittled by baking. These supported (2) over (1) (refer original proposal). Subsequently a more comprehensive set of tests on scaled single-edge-cracked specimens made of the same model material have been undertaken and reported in Kondo and Sinclair [1] (copy attached as Appendix 1). In [1], the material is demonstrated to be extremely brittle in good accord with the assumptions underpinning elastic fracture mechanics, and some 244 individual tests are involved enabling the effects of scatter to gauged and controlled. The results found are summarized below.

Table 1. Fracture stress ratios from [1]

Scale factor, λ	Mean ratio, $\overline{\sigma_1^* / \sigma_2^*}$	95% confidence limits
1.5	1.011	0.928-1.094
2.0	0.994	0.923-1.065
3.0	1.001	0.910-1.092
4.0	1.047	0.994-1.100

These results demonstrate statistically significant differences between actual physical behavior and that predicted by LEFM, (1), while admitting the possibility that (2) holds within scatter.

Discussion of these results with members of the fracture mechanics community [2] lead to our being directed towards references containing data such that the converse was true, i.e. (1) was supported over (2). The most convincing of such references was the paper by Lubahn and Yukawa [3], and in particular, Irwin's discussion [4] of the same. There, for a wide range of scale factors, very good agreement with LEFM (1) is exhibited, although the specimens are not perfectly scaled with respect to notch acuity. Too, the response for the most part is quite ductile with the yield region typically being considerably in excess of 2% of the crack length, the limit usually regarded as demarking brittle from ductile behavior (cf. ASTM standards for plane strain fracture toughness testing [5]). This contrasting behavior nonetheless motivated two activities: carrying out our own set of tests on a more ductile material and simultaneously performing an extensive literature search for data bearing on the issue.

The results of the first activity for aluminum sheets are presented in detail in Keremes and Sinclair [6] (attached as Appendix 2). In summary, the 60 double-edge-cracked specimens gave:

$$\lambda = 3, \ \overline{\sigma_1^\star/\sigma_2^\star} = 1.014, \ 95\% \text{ confidence limits} = 0.998\text{-}1.029.$$

Thus a physical demonstration of support for (2) over (1), the opposite to the trends reported in [3]. Clearly the situation does not have one simple answer.

An answer of sorts is furnished by the review of some 300 references containing strength size effects data (see bibliography [7], copy attached as Appendix 3). From these references, 100 odd independent sources can be drawn which contain strength dependencies for truly in-plane scaled cracked specimens (with thickness effects controlled). The data from these pertinent references are analyzed at some length in Sinclair and Chambers [8] (Appendix 4). In essence the data show that neither (1) nor (2) holds, irrespective of whether the tests are for very brittle response or quasi-brittle response, for plane strain or plane stress, for valid K_{IC} testing or not, etc. Occasionally the size independent response (2) is found:

far more often there are microstructural size effects so that there is a reduction in strength with size, the same trend as in (1), yet seldom are such effects even roughly tracked by this simple formula. The situation is too complex for such naive predictions (1), (2) of size dependence (independence) to be appropriate.

A partial explanation as to what is going wrong in the classical arguments that linear elastic fracture mechanics is founded on - essentially the thermodynamic argument of Griffith [9] - is given in Sinclair [10] (Appendix 5). A possible explanation of the data itself, based on an extension of that first given by Weiss and Yukawa [11], is given in Sinclair [12] (Appendix 6). Both [10], [12] are at a stage that might best be termed ongoing research at this time. Their full development was not intended to be a part of the one-year research program for which this is the final report, and thus it is not appropriate to discuss them in any detail here. What can be said, however, is that size effects are almost certainly due to microstructure and consequently can be expected to differ from one material to the next, even when response is brittle. Further, for a specific material, they can depend on size itself, typically decreasing with increasing size. Thus there is no one formula like (1) with a single exponent (there one half), but a variety of different exponents which are both material and geometry dependent themselves. Given this far more complex character, (1) is quite inadequate, and the idea that (2) applies unless there are microstructural effects probably true but largely useless. We offer some concluding remarks in the light of these observations next.

CLUCLUDING REMARKS

The prediction of strength size effects in fracture mechanics using either LEFM or DEFM is so naively simple as to be manifestly incomplete. Accordingly both can lead to predictions that are inaccurate to the point of not being acceptable in engineering. Moreover such errors are not necessarily conservative. For example, LEFM is typically nonconservative when testing a specimen larger than the size of the intended application, a situation that can certainly arise in practice, and DEFM is nonconservative in reverse circumstances.

In practice, then, it is preferable, if not necessary, to test on the same size scale as the application. In the event of this being impractical, the following strategy might be adopted. For the most part, the size effects predicted by LEFM are in excess of those for the actual data. Hence when testing small and applying big, LEFM can be used to estimate the reduction in strength and usually will do so conservatively. On the other hand, when testing big and applying small, the LEFM prediction can greatly exceed the strength increases in fact realized. Here, though, it would appear that strength seldom decreases with decreasing size. Consequently, using DEFM will generally be conservative.

A caution on the use of the above is in order. There is really no physical reasoning underlying the scheme; it is merely based on observation of the data. And this data is not always confined within LEFM's prediction of size effects and the size independence of DEFM, so that there is physical evidence of the strategy being nonconservative. Some judgement is therefore required in implementing this essentially empirical approach.

On a more fundamental front, there is reason to be concerned about the very basis of fracture mechanics. These concerns arise because there exist several hundred test results for appropriately brittle behavior *not* agreeing with the LEFM prediction of strength size effects (see [8]). Every one of these represents data establishing a variation in fracture toughness with size. It follows that fracture toughness is demonstrably *not* a material property. Thus the use of the stress intensity factor as the parameter in and of itself controlling brittle fracture needs serious examination.

Acknowledgements - We are pleased to acknowledge the valuable benefit of discussions during the course of this work with J.H. Griffin of the Department of Mechanical Engineering at Carnegie-Mellon University and with D.A. Glasgow of the Air Force Office of Scientific Research (AFOSR). The financial support of AFOSR is also appreciated.

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APPENDIX 1

STRENGTH SIZE EFFECTS FOR EMBRITTLED PAPER SPECIMENS WITH CRACKS AND SEMICIRCULAR NOTCHES IN A SINGLE EDGE

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Report SM 85-5

March 1985

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STRENGTH SIZE EFFECTS FOR EMBRITTLED PAPER SPECIMENS
WITH CRACKS AND SEMICIRCULAR NOTCHES IN A SINGLE EDGE

INTRODUCTION

Strength size effects occur when two geometrically similar configurations comprised of the same material but having different sizes fracture at different applied stress levels. Such effects must be limited if the inferred physical measures of stress increase due to the presence of cracks and sharp re-entrant corners in Sinclair and Kondo [1] are to be meaningful. The primary objective of the work reported here is to check for size effects in the model brittle material used in [1].

In this connection we note that there are theories which predict size dependence in *brittle* materials. For *cracked* geometries, *linear elastic fracture mechanics* (LEFM) has

$$\sigma_1^*/\sigma_2^* = \sqrt{\lambda}, \tag{1}$$

wherein σ_i^* $(i = 1,2)$ are the applied stresses at fracture in two similar cracked specimens, the second of which has its in-plane dimensions increased by the scale factor λ. Hence LEFM predicts a specific reduction in strength with increase in size for all brittle materials with cracks. For *general* geometries, *Weibull's statistical approach* [2] leads to

$$\sigma_1^*/\sigma_2^* = (V_1/V_2)^{-1/m}, \tag{2}$$

where V_i is the stressed volume for Specimen i $(i = 1,2)$, and m is a material parameter. For $m > 0$, this also gives rise to a reduction in strength with increasing size, but now different rates of reduction are admitted including the possibility of size independence $(m \to \infty)$. Thus there would seem to be a real need to check what size dependence if any exists for the model brittle material of [1], especially for cracked specimens.

This report describes three different sets of experiments carried out to this end during the course of the last two years. The first of these examined fracture stress ratios of single-edge cracks with scale factors of $\lambda = 1.5, 2, 3, 4$ (performed in June, 1983), the second concerned more extensive testing of the extreme ($\lambda = 4$) of the first set (October, 1983), and the third looked at a control set of single-edge semicircular notches with $\lambda = 4$ (April, 1984). In what follows we begin by discussing the experimental setup for all three sets in Section 1, then close by presenting attendant results in Section 2 (complete experimental data are furnished in the Appendix).

1. EXPERIMENTAL DESIGN

The model material employed in Sinclair and Kondo [1] is baked Xerox paper. Details of the preparation of this embrittled paper and the fabrication and inspection procedures for specimens made out of it are given in [1], together with the reasons for its use, so that here we merely summarize the key aspects.

This model material has the attribute of readily enabling fabrication of specimens at low cost and with little effort. Suitably heat treated it is very brittle, thereby approximating well the linear elastic response up to the point of fracture sought in [1]. Further its thinness ensures a two-dimensional stress state but it is still sufficiently thick so as to prevent buckling. In all this material represents good compliance with the properties sought in [1], yet also should be one for which LEFM is most applicable.

The embrittled paper specimens for testing size effects are shown in Fig.1 and have actual dimensions as in Table 1 (all are 9 inches long and 0.0035 inches thick). Sets of each specimen type are prepared, inspected for surface nicks or burrs, and checked for eveness of baking. Any not meeting standards are discarded *before* testing. The remainder are pulled in a calibrated Hounsfield tensometer and *all* fracture stress ratios recorded. The sample sizes tested are 77, 30, 30,

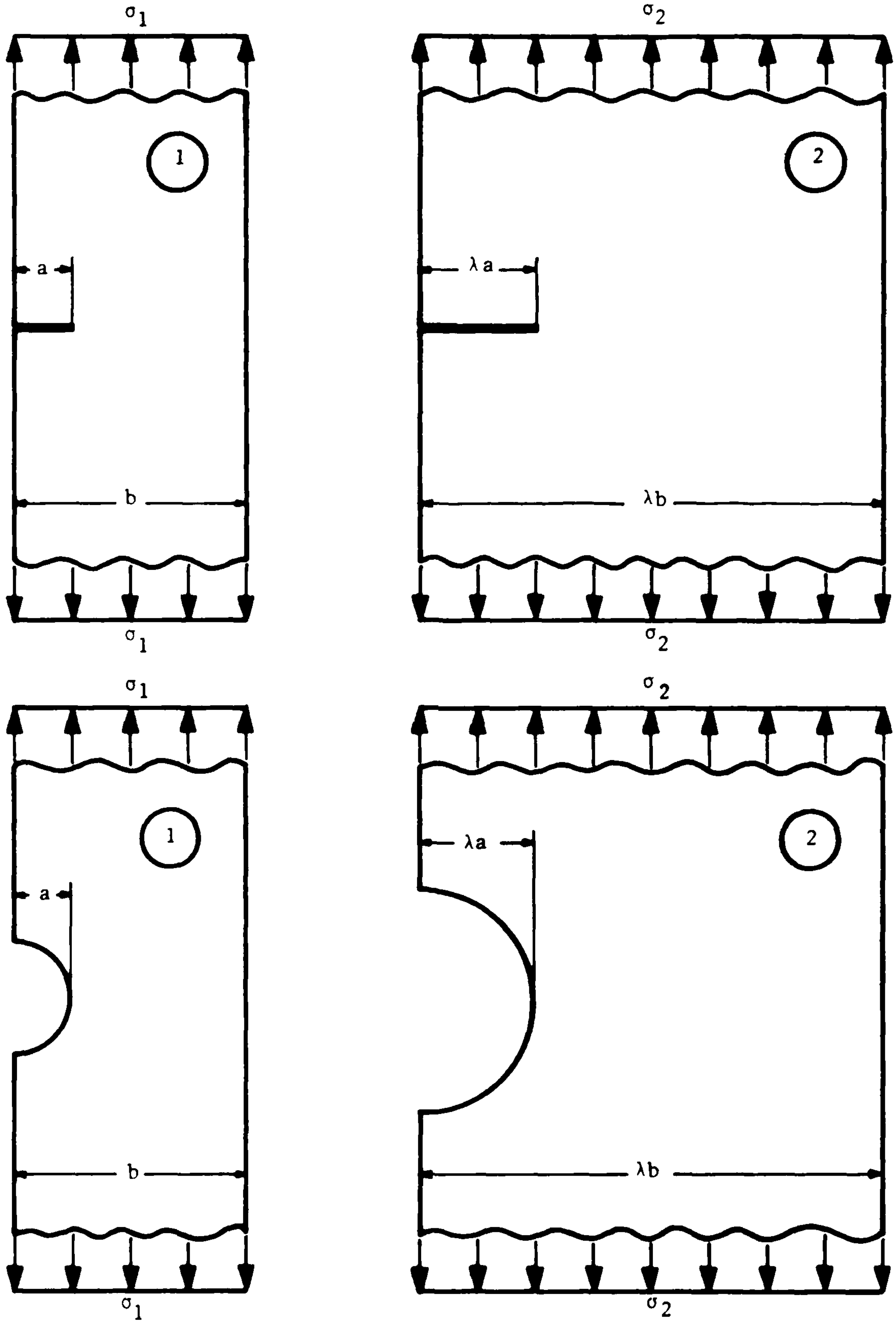

Fig. 1. Scaled specimen pairs

30, 77 for λ = 1, 1.5, 2, 3, 4 respectively for the cracked specimens, and 38 for both λ = 1 and λ = 4 for the semicircular notch specimens. Provided approximately normal distributions of the fracture stress ratios result, sample sizes of 30 or larger enable the use of the Central Limit Theorem (see, e.g. Wine [8]; as a check on this requirement histograms of the data are examined). Then the 95% confidence limits in the *mean* fracture stress ratio, $(\overline{\sigma_1^*/\sigma_2^*})$, can be approximated by $(\overline{\sigma_1^*/\sigma_2^*})$ $\pm\ 1.96s/\sqrt{N}$, where s is the *sample* standard deviation in σ_1^*/σ_2^* and N is the sample size. In this way it is hoped to control the scatter sufficiently to resolve the issue at hand.

Table 1. Specimen dimensions (in inches)

Specimen type	Scale factor, λ	Crack length/ notch depth, a	Width, b
Crack	1.0	0.2	0.8
Crack	1.5	0.3	1.2
Crack	2.0	0.4	1.6
Crack	3.0	0.6	2.4
Crack	4.0	0.8	3.2
Notch	1.0	0.3	1.2
Notch	4.0	1.2	4.8

2. RESULTS

The results for the fracture stress ratios for the different specimens are summarized in Table 2. These show that the mean fracture stress ratios differ from unity by no more than 5% and typically by around 2%. The scatter in the experimental ranges is of the order of ± 45% about the mean with a maximum deviation of 62% for the cracked specimens, while the range for the notched specimens is close to ± 20%. However, since the data are in fact approximately normally distributed (see Appendix), the Central Limit Theorem can be invoked to in effect restrict scatter to about ± 8% for λ = 1.5, 2, 3 and ± 5% for λ = 4 for the cracked specimens, and to about ± 3% for the notched specimens. Accordingly the results represent a solid demonstration of *strength size insensitivity* for the model material of [1], even for cracked instances.

Table 2. Fracture stress ratio results

Specimen type	Scale factor λ	Mean ratio $\overline{(\sigma_1^*/\sigma_2^*)}$	Experimental range	95% confidence limits
Crack	1.5	1.011	0.612-1.539	0.928-1.094
Crack	2.0	0.994	0.680-1.513	0.923-1.065
Crack	3.0	1.001	0.722-1.622	0.910-1.092
Crack	4.0	1.047	0.617-1.536	0.994-1.100
Notch	4.0	1.015	0.851-1.250	0.983-1.047

Comparing the outcomes represented in Table 2 with the LEFM prediction (1), we see, that despite the physical correspondence to the underlying assumptions in elastic fracture mechanics, the data here are in clear disagreement. More precisely, (1) has σ_1^*/σ_2^* equal to 1.22, 1.41, 1.73, 2.00 for λ = 1.5, 2, 3, 4 whereas experimentally we find $\overline{(\sigma_1^*/\sigma_2^*)}$ of 1.01, 0.99, 1.00, 1.05 respectively. Moreover, for the last two scale factors, not even a single outlier in over 100 tests attained the value predicted by LEFM, while for all λ, the experimental 95% confidence limits excluded the σ_1^*/σ_2^* of (1).

Turning to the Weibull model of strength size effects and fitting (2) to to the mean values in Table 2 yields m = 37, -115, 1099, 30 for λ = 1.5, 2, 3, 4 for the cracked specimens and m = 93 for the notched specimens. These large and varying values are typical of those found when strength response is size independent to all intensive purposes and underscore the disagreement with elastic fracture mechanics here (m = 2 in LEFM for the present geometries).

In sum, the strength size *independence* of the given model material required for the study in Sinclair and Kondo [1] to be useful would appear to exist. This size independence differs markedly from the size *dependence* for brittle materials implicit in linear elastic fracture mechanics.

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APPENDIX

Here we give the actual experimental data for the fracture stress ratios (Table 3) and histograms of the results for the cracked specimens (Fig. 2). The results for the notched specimens conform more closely to a normal distribution than those in Fig. 2.

Table 3. Fracture stress ratios, σ_1^*/σ_2^*

Cracked specimens						Notched specimens	
$\lambda = 1.5$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$	$\lambda = 4$	$\lambda = 4$	$\lambda = 4$	$\lambda = 4$
1.195	0.966	1.102	1.092	1.406	1.148	1.047	1.000
0.846	1.085	1.000	0.909	1.148	1.087	1.115	1.070
0.832	0.852	0.813	0.877	1.508	1.359	0.982	0.991
1.226	0.680	0.920	0.935	1.042	0.617	1.041	1.004
0.976	0.878	0.924	0.735	0.840	1.160	1.250	0.896
0.942	1.513	0.850	0.847	0.714	0.794	1.130	0.903
0.984	1.353	1.622	1.385	0.855	0.990	1.056	0.851
1.139	0.810	1.158	1.012	1.033	1.282	0.986	0.912
1.539	1.045	1.130	0.855	1.103	1.022	0.919	
0.838	0.852	0.784	1.364	0.952	0.645	0.979	
1.400	1.121	1.130	1.304	1.307	1.172	1.005	
1.202	1.057	1.021	1.458	1.248	0.787	0.983	
0.790	1.100	0.863	1.267	0.935	0.813	0.949	
1.000	1.245	0.769	1.536	0.820	0.909	1.045	
0.865	0.934	1.255	1.508	0.870	0.621	1.082	
1.157	1.042	1.157	0.952	1.250	0.877	1.169	
0.952	0.785	0.863	1.368	1.364	1.214	1.132	
1.046	1.103	0.925	1.104	1.344		1.000	
0.747	1.000	0.722	1.000	0.847		1.088	
0.833	0.851	0.851	1.000	1.211		0.867	
0.612	0.703	0.722	1.250	1.000		0.936	
1.392	1.243	1.101	0.820	1.043		1.153	
0.924	0.859	0.913	0.840	1.250		0.852	
0.899	0.707	0.869	0.862	0.758		0.991	
1.318	1.101	1.865	0.769	0.654		1.021	
0.874	1.115	0.954	0.909	1.285		1.044	
0.616	0.718	0.735	1.269	0.909		0.969	
1.215	1.106	1.253	0.787	1.309		0.868	
0.815	0.930	0.902	1.211	0.917		1.247	
1.144	1.080	0.865	1.124	0.847		1.054	

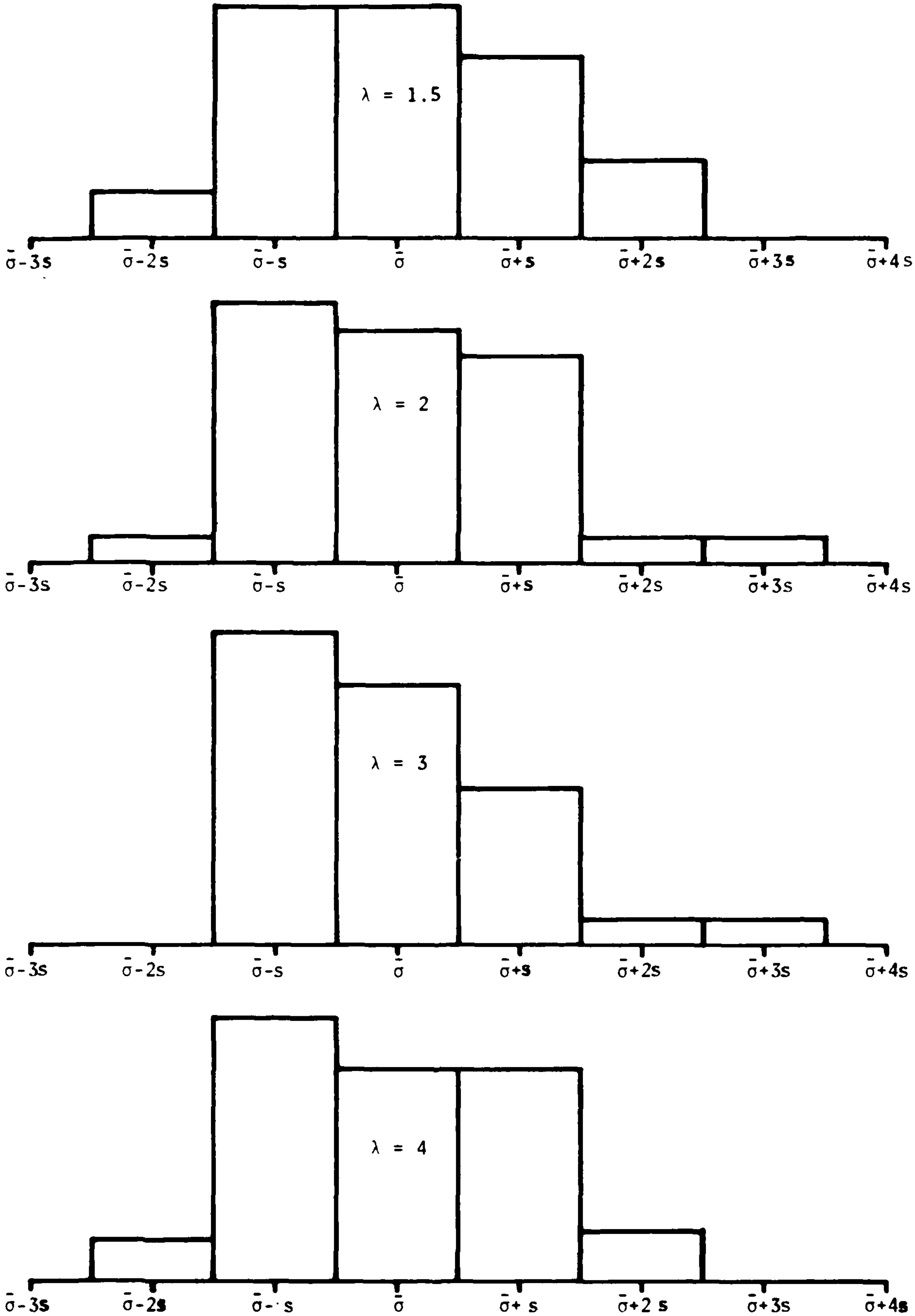

Fig. 2. Relative frequencies of fracture stress ratio data for the cracked specimens ($\bar{\sigma} = \overline{(\sigma_1^*/\sigma_2^*)}$)

APPENDIX 2

A PILOT EXPERIMENTAL STUDY OF SIZE EFFECTS IN THE FRACTURE OF DOUBLE-EDGE-NOTCHED ALUMINUM SHEET SPECIMENS

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Report SM84-26

November, 1984

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A PILOT EXPERIMENTAL STUDY OF SIZE EFFECTS IN THE FRACTURE OF DOUBLE-EDGE-NOTCHED ALUMINUM SHEET SPECIMENS

INTRODUCTION

Size effects occur when two different sizes of specimens having the same shape, material, and loading configuration, fracture at different nominal stresses. In this connection, linear elastic fracture mechanics (LEFM) implies that the ratio of nominal stresses at fracture between two cracked specimens is equal to the square root of their scaling factor (λ), the so-called *geometric size effect*. Thus

$$\frac{\sigma_1^*}{\sigma_2^*} = \lambda, \tag{1}$$

where σ_1^*, σ_2^* are the nominal stresses at fracture in Specimens 1, 2 respectively, with Specimen 2 being λ times as big as Specimen 1. Alternatively, a simple Weibull model [1] based on a statistical theory of fracture has that

$$\frac{\sigma_1^*}{\sigma_2^*} = \left(\frac{V_1}{V_2}\right)^{-1/m} (m > 0), \tag{2}$$

where V_1, V_2 are the respective volumes of Specimens 1, 2, and m is a material parameter found by experimentation. Equation (2) represents a means of fitting *microstructural size effects.* Both (1) and (2) are in agreement with the general trend found in actual data, namely that strength decreases with size. However, there are significant differences between these two: (1) has a fixed dependence for all materials with this dependence being on in-plane dimensions alone, whereas (2) has a size dependency which is material sensitive and which exhibits variations with both in-plane and out-of-plane dimensions. The basis objective here is to experimentally examine for selected configurations whether (1) holds, (2) holds, both hold or neither.

Generally one expects the LEFM prediction, (1), to apply best when the response is brittle. Other components of the research program here at C-MU examine the applicability of (1) in this case. However, there exists claims in the literature (see, for example, Irwin's discussion of Lubahn and

Yukawa [2]) which suggest that (1) holds even with ductile response. As a result we seek to examine the validity of (1) here for a material which exhibits some ductility. There is an extensive literature on this subject and a comprehensive review falls outside the scope of this report. Nonetheless, it would be fair to say even on the basis of a limited review [3-7], that generally size effects do occur in ductile materials with significant stress concentraters, but that these effects do not necessarily adhere to the specific size dependence of (1). This last though may be attributable to scatter in the results. As a consequence a second aim of this experimental study is to control scatter sufficienty to resolve the issue as to which of (1), (2) is most appropriate.

In what follows we first describe a set of experiments designed to meet our two objectives in Section 1, then present the attendant results in Section 2.

1. EXPERIMENTAL DESIGN

In this section we describe the reasons underlying our material selection and the manner in which the tests are performed.

The selection of testing material is governed by the criteria of exhibiting significant ductility and enabling reproducibility. A suitable material should thus have a plastic strain to elastic strain ratio which is at least two at rupture in a uniaxial tension test. In addition, an appropriate material must also allow reproducible results, either by having little inherent scatter or by readily permitting a sufficiently large set of experiments to be performed which will in effect limit scatter.

One material which has the potential of satisfying the above criteria is aluminum in the form of thin sheets. This material can be expected to exhibit some significant ductility; simple tensile tests on straight specimens serve to check this expectation. The thinness of the sheet material, moreover, makes it easy, both from an effort and an economics viewpoint, to manufacture a large number of specimens.

Unfortunately, when obtaining standard sheets it is not generally possible to scale the thickness of the specimens in unison with the rest of the dimensions. As a result we choose to use a single sheet thickness and thereby reduce variations from one manufactured roll to another. Provided this thickness is small enough, a two-dimensional state of stress (plane stress) should still be induced. Observe that when thickness is not scaled, the possibility of buckling is increased with larger in-plane dimensions; thus, the largest specimen is the critical specimen in which to prevent buckling. To minimize the possibility of buckling in the largest specimen, the thickest freely available commercial sheet is chosen which has a thickness of 0.00093 inches. This selection serves to sufficiently limit buckling here and is still 10^3 times smaller than the smallest of the in-plane dimensions so that a two-dimensional geometry is obtained.

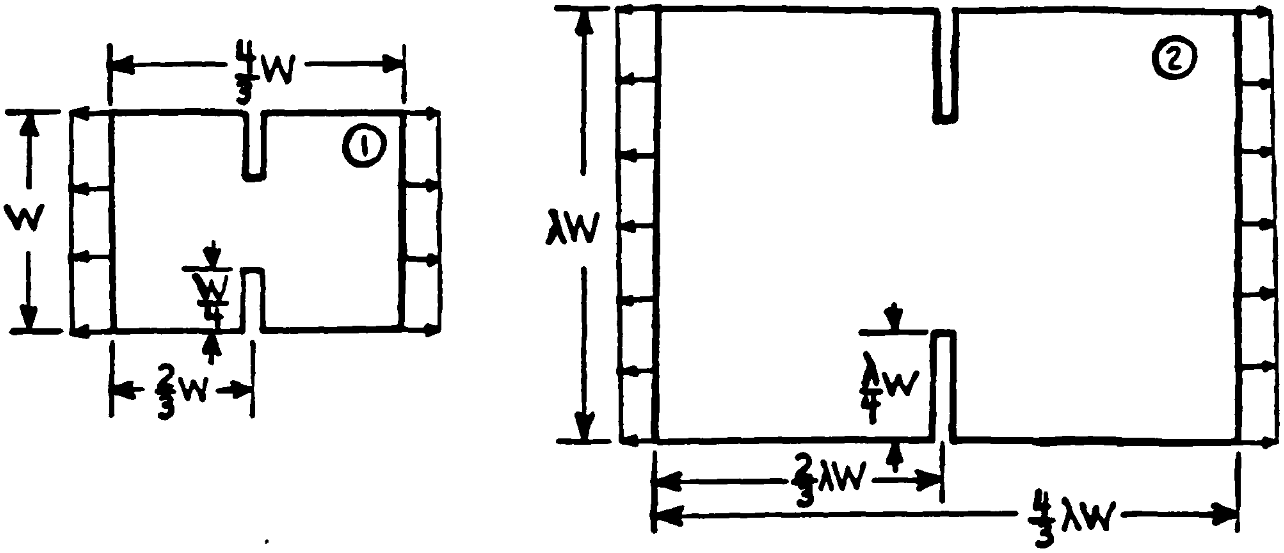
A Hounsfield Tensometer with a light beam spring is used to test the specimens. A checked spring balance is used to calibrate the force measurements of the tensometer. Fine sandpaper placed on the testing grips prevents the specimens from slipping. The grips themselves slide on bars which are dusted with powdered chalk to reduce friction.

A geometric scale factor of 3:1 ($\lambda = 3$) is chosen for the specimens since their size range is restricted by the testing equipment. This scale factor though is large enough to provide observable results between the two sizes of specimens should a size effect exist. Three different shapes of notched specimens are tested (Fig. 1). The two circularly notched specimens serve as a reference for the key geometry in the experiments which is the cracked configuration pair: the sharply notched specimens provide an intervening geometry. The specimens are all double-edge-notched to prevent the bending which would occur in single-notched specimens.

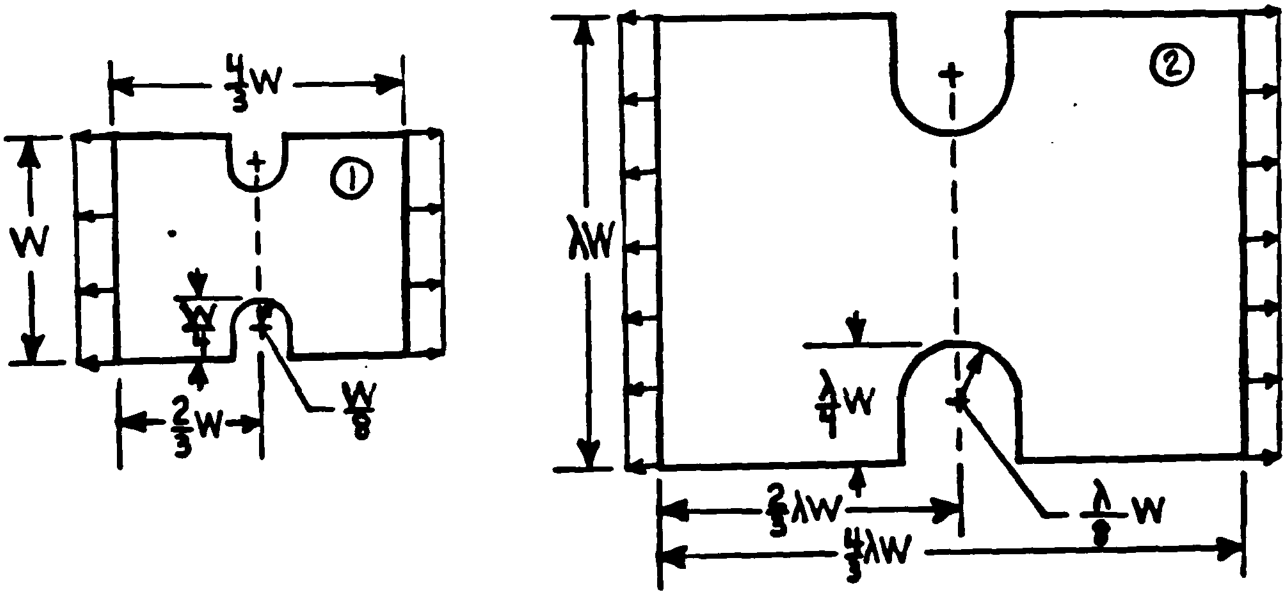
When making the specimens the longest dimension is always parallel to the rolling direction of the aluminum foil. After being manufactured the specimens are examined for defects. All of the specimens judged to be defective are removed from the specimen pool *prior* to testing. The ends of the accepted specimens are then placed into the grips of the tensometer and loaded until fracture occurs. A total of 30 specimens are tested in the tensometer for each particular size and shape of specimen in Fig.1 and *all* results found included. This number of tests enables the use of the central limit theorem which recognizes that more measurements enable us to know the value of the mean better. The theorem in effect states that the standard deviation *in the mean* is equal to the population standard deviation divided by the square root of the sample size. Given a sufficiently large sample size we can reasonably approximate the population standard deviation by s, the *sample* standard deviation: then the approximate 95% confidence limits for the mean fracture stress, $\bar{\sigma}^*$, are $\bar{\sigma}^* \pm 1.96s/\sqrt{N}$, where N is the sample size. It is generally accepted that "sufficiently large" in practice means $N \geq 30$ providing that the distribution is not of an unusual shape (see Wine [8]). Histograms are used to check this last point.

2. RESULTS

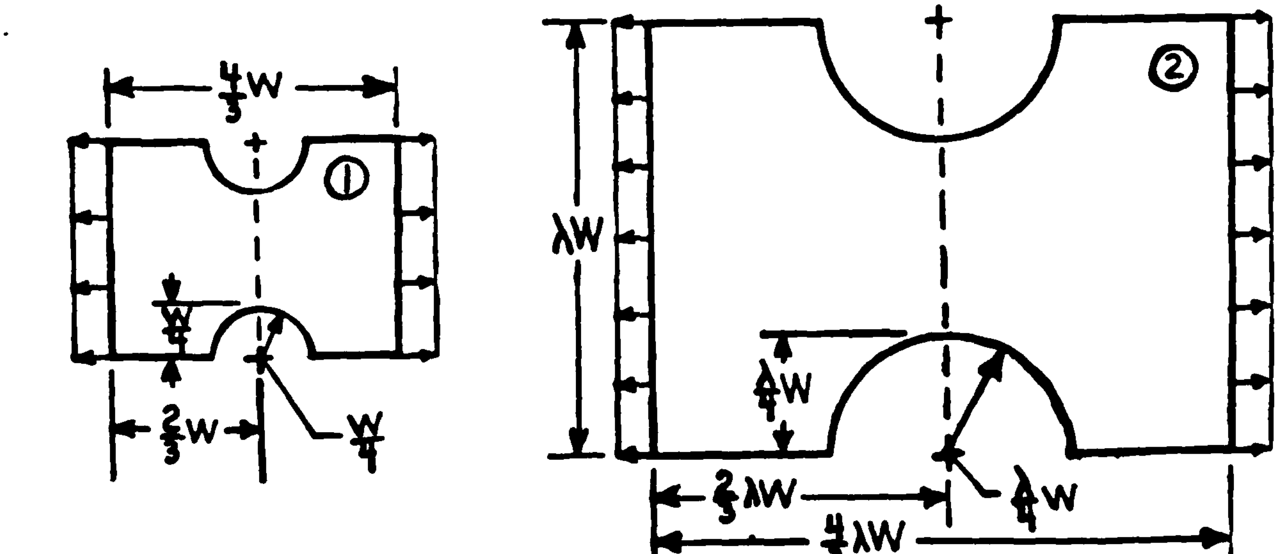
Here we first present our results and then comment on their relation to existing theories.



Double edge crack specimens



Double edge deep circular notch specimens



Double edge semicircular notch specimens

Fig. 1 Scaled test specimens (w = 4.50 in.)

A uniaxial stress (σ) versus strain (ϵ) curve for unnotched straight aluminum sheet specimens is shown in Fig.2. Here σ, ϵ have been normalized by their respective values at yield, σ_Y, ϵ_Y. This graph has a ratio of 6.3 for the strain at fracture (ϵ_f) divided by the yield point strain (ϵ_Y). This value is less than that for bulk aluminum because of the cold-work involved in sheet production and typically less than values for aluminum alloys and mild steel: nonetheless it represents a marked degree of ductility and we can expect the net section stress to exceed the yield stress in our notched specimens - the hallmark of significantly ductile response for such configurations. Indeed this is found to be the case for all notched specimens with typically the ratio of nominal stress to yield stress being 1.8 and ranging from 1.67 to 1.91.

With the thickness used buckling as expected is limited. Only the largest specimens exhibit perceptible buckling. This occurs at the ends of the specimens near the grips in the form of a sinusoid of the order of 3 cycles with an amplitude of about 0.5% of the width. There is buckling in the vicinity of any of the notches.

The results of the notched tensile tests are summarized in Table 1. The results show that all three specimens are basically size independent. While the scatter reflected in the actual untreated experimental ranges might permit an appreciable size effect to be present yet remain undetected, the 95% confidence limits indicate no such possibility with size effects restricted to no more than 3%. These 95% confidence limits are based on the central limit theorem, the use of which is justified by the histograms in the Appendix.

Table 1. The effect of size on the mean fracture stress ratio.

Specimen	$(\sigma_1^* / \sigma_2^*)$	Raw experimental range	95% confidence limits
Double-edge crack	1.014	0.924-1.100	0.998-1.029
Double-edge deep circular notch	1.010	0.932-1.139	0.992-1.027
Double-edge semi-circular notch	1.008	0.903-1.096	0.989-1.026

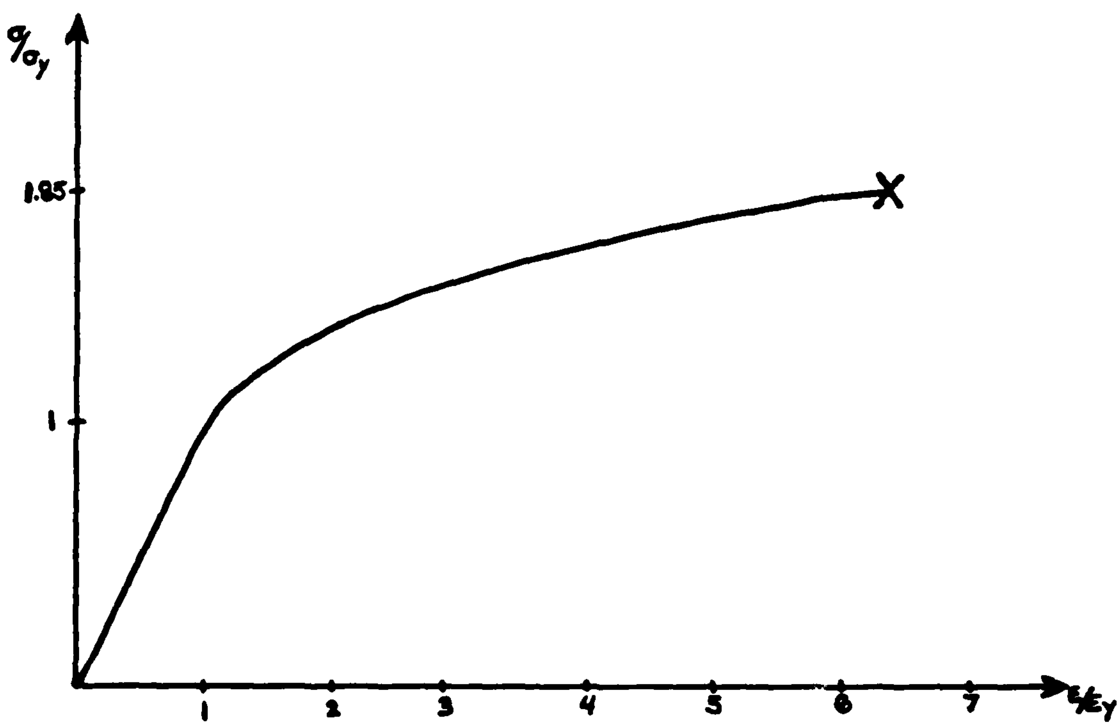


Fig. 2 Stress-strain data for unnotched aluminum sheet

To all intensive purposes, the three types of specimen are size independent. Notched pure aluminum bars tested in Klier and Weiss [9] also show no noticeable size effect which is consistent with the present data. The results do indicate a small increase in the fracture stress with a decrease in size. As remarked earlier, an increase in strength with decrease in size is consistent with other size effect phenomena. It is interesting to note that although the size effect is small it is similar for all three types of notches.

Although we have ductile response here so that LEFM is not appropriate, we can compare with the LEFM prediction (1) to see if it extends into the ductile region as has been suggested by other investigators on occasion. Clearly (1) cannot in general admit to such an extension since here we demonstrate that it is violated, and statistically significantly so. Moreover, the presence of size effects of type (2) cannot alleviate this contradiction.

Applying the simple Weibul model (2) to the three specimens gives a range of values for m from 148-276. The high values of m show that the specimens are practically size insensitive. It should be noted that the value of m with such small size effects is uncertain since in this range of size effects the value of m can change appreciably with a small percentage change in the σ_1^* / σ_2^* ratio. Even so, this value is markedly different from that contained in effect in LEFM (ie. an m of 2 here), underscoring that the LEFM size prediction does not apply here.

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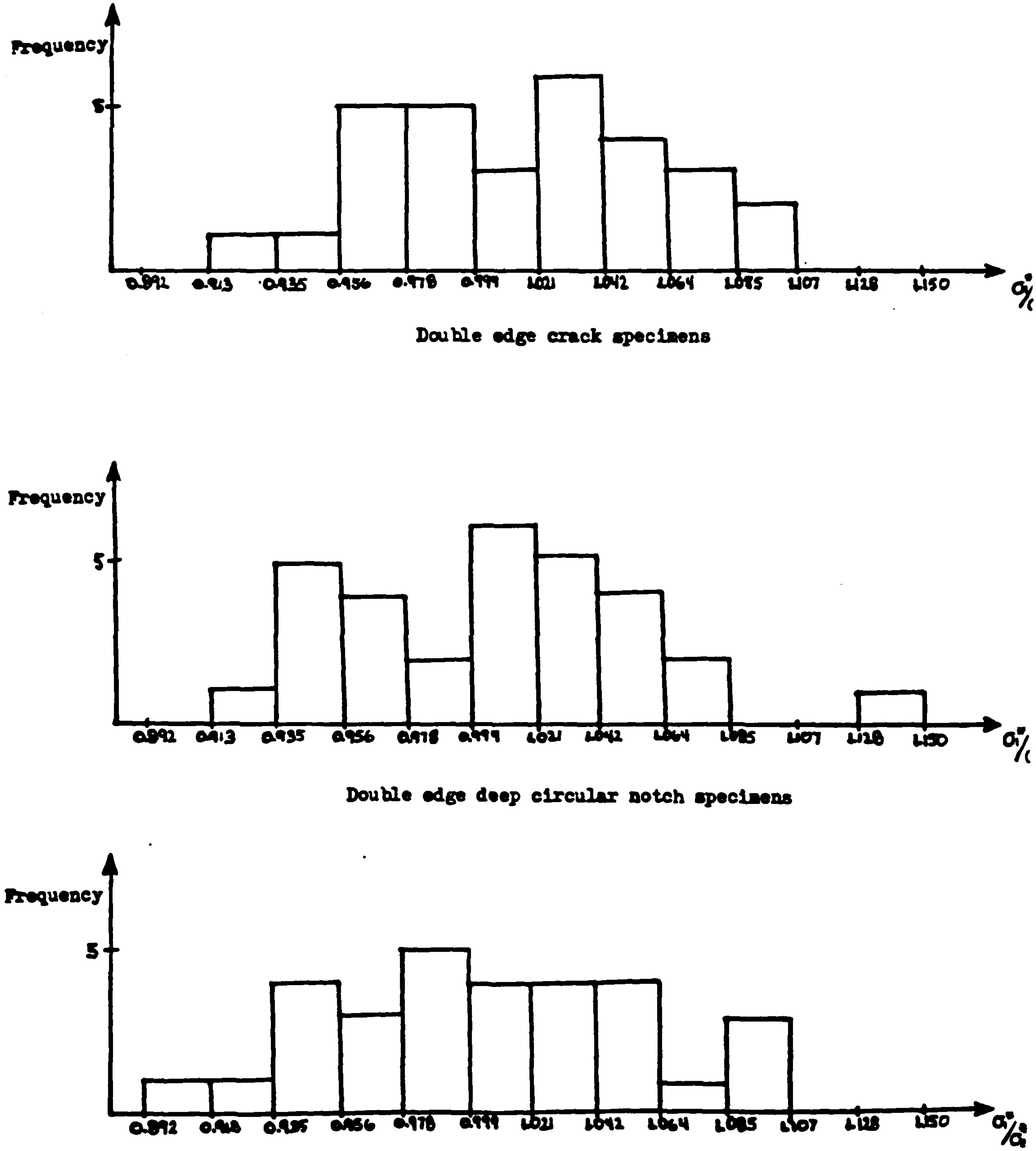
Appendix

Here we furnish the details of the experimental data.

Table 2. < Strength ratios (σ_1^*/σ_2^*) for different specimens shapes.

Specimen number	Double-edge crack specimens	Double-edge deep circular notch specimens	Double-edge deep semicircular notch specimens
1	0.950	1.048	1.017
2	1.038	1.042	0.990
3	0.924	1.016	1.094
4	1.091	1.139	1.032
5	1.025	0.977	1.042
6	1.001	0.938	1.058
7	1.025	1.034	1.073
8	0.979	1.034	1.058
9	1.017	1.015	0.972
10	1.066	1.077	0.929
11	1.016	0.973	0.990
12	1.071	1.044	1.018
13	0.976	0.951	1.087
14	1.018	1.064	1.056
15	1.064	1.008	0.995
16	1.022	0.932	1.096
17	0.961	0.991	1.000
18	0.976	0.947	0.961
19	0.984	0.952	0.952
20	1.028	0.951	1.037
21	0.987	1.021	0.938
22	1.049	1.002	0.903
23	1.028	1.018	0.952
24	0.958	1.076	1.020
25	1.061	1.063	1.021
26	0.976	1.027	0.967
27	1.100	0.973	1.037
28	0.979	0.968	0.998
29	0.993	1.030	0.951
30	1.043	0.981	0.986



Double edge crack specimens

Double edge deep circular notch specimens

Double edge semicircular notch specimens

Fig. 3 Histograms of the strength ratios in Table 2

Histograms of the data in Table 2 are presented in Fig.3 as evidence that the data approximate a normal distribution permitting the use of the central limit theorem. As further evidence, the standard deviations for the first 15 and second 15 specimens are calculated separately to see if they are numerically consistent with asymptotically approaching a limiting value. These results are given in Table 3 and support the use of the sample standard deviation as an estimate of the corresponding population values.

Table 3. Means and standard deviations of the experimental data in table 2.

Source	Mean	Standard deviation
Double-edge crack		
1st 15 specimens	1.017	0.0464
2nd 15 specimens	1.010	0.0414
All 30 specimens	1.014	0.0434
Double-edge deep circular notch		
1st 15 specimens	1.024	0.0515
2nd 15 specimens	0.995	0.0432
All 30 specimens	1.010	0.0489
Double-edge semicircular notch		
1st 15 specimens	1.027	0.0461
2nd 15 specimens	0.988	0.0490
All 30 specimens	1.008	0.0509

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Nominal Breaking stress at minimum width — 1½ by 2 inch Specimens

Specimen No.	Double Crack (lb/in^2)	Double 1⅛ by 1⅛ (lb/in^2)	Double ½ circle (lb/in^2)
1	6038	6796	6850
2	6201	6905	6742
3	6038	6958	7013
4	5875	7284	7068
5	6254	6688	7094
6	6146	6525	7338
7	6200	7175	7500
8	6011	6742	7067
9	6200	6417	6742
10	6255	7175	7392
11	6309	6823	6959
12	6363	6959	7121
13	6255	6580	7500
14	6417	7013	7230
15	6417	6959	7230
16	6257	6752	7246
17	6092	6642	7137
18	6257	6587	6917
19	6092	6642	6862
20	6422	6752	7247
21	6092	6807	6697
22	6422	6862	6807
23	6477	6972	6917
24	6037	7192	7357
25	6587	7082	7192
26	6202	6972	6972
27	6532	6752	7137
28	6257	6752	7192
29	6147	6807	7027
30	6532	6752	6752

Nominal breaking stress at minimum width

4 1/2 by 6 inch Specimens

Specimen No.	Double Crack $\left(\frac{lb}{in^2}\right)$	Double 1 1/8 by 1 1/8 $\left(\frac{lb}{in^2}\right)$	Double Half Circle $\left(\frac{lb}{in^2}\right)$
1	6356	6484	6738
2	5975	6629	6811
3	6538	6847	6411
4	6411	6393	6847
5	6102	6847	6811
6	6138	6956	6938
7	6048	6938	6992
8	6138	6520	6682
9	5787	6320	6938
10	5866	6665	6865
11	6211	7010	7029
12	5939	6665	6992
13	6411	6920	6901
14	6302	6593	6847
15	6029	6901	7265
16	6120	7247	6611
17	6338	6702	7138
18	6411	6956	7119
19	6193	6974	7210
20	6247	7101	6992
21	6175	6665	7138
22	6120	6847	7537
23	6302	6847	7265
24	6302	6683	7210
25	6211	6665	7047
26	6356	6792	7210
27	5939	6938	6883
28	6393	6974	7210
29	6193	6611	7392
30	6266	6853	6847

APPENDIX 3

A BIBLIOGRAPHY OF STRENGTH SIZE EFFECTS FOR CRACKED SPECIMENS

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SM 85-10 May 1985

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A BIBLIOGRAPHY OF STRENGTH SIZE EFFECTS FOR CRACKED SPECIMENS

PREFACE

The references gathered here are with a view to providing strength test data for cracked or sharp notch specimens which have their dimensions in the plane of the crack scaled. The first table lists all sources with such in-plane scaling which are used in a survey of this type of data*: the remaining tables are all references with size-effect strength data consulted in the course of compiling Table I but not included therein for various reasons. For Table II, most references either could not be shown to have *all* dimensions effectively scaled or presented data already contained in the references of Table I. For Table III, most results in references were affected to some extent by out-of-plane or thickness effects, since either specimens did not have their thicknesses scaled in concert with their in-plane dimensions or did not consistently maintain a state of plane stress or strain (there are some references, however, which are also listed in Table I because they had both in-plane and thickness size effects). For Table IV, all sources either involved specimens with notches whose acuity was not judged to be sufficient to qualify as cracks or entailed specimens with no notches whatsoever. It should be emphasized that neither Table III nor Table IV approach being comprehensive surveys of their data types - they are simply the test results of their respective natures encountered in assembling Table I. Each table is arranged alphabetically by author (s).

Acknowledgements - The assistance of A. Chambers and R. Pieri in checking citations in this bibliography is much appreciated. The financial support of this study by the Air Force Office of Scientific Research is gratefully acknowledged.

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STRENGTH SIZE EFFECTS AND FRACTURE MECHANICS:
WHAT DOES THE PHYSICAL EVIDENCE SAY?

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Abstract - Being able to predict the strength of geometrically-similar cracked specimens of different sizes or scales is a fundamental requirement for success for linear elastic fracture mechanics (LEFM). The prediction contained in LEFM is that the strength reduces as the inverse square-root of the scale factor in the plane of the crack: here we review how well this prediction actually agrees with the physical evidence. In particular we examine agreement for materials and configurations exhibiting brittle responses - the situations complying best with the underlying linear elastic assumptions in the theory. The data shows that the agreement is not good, even in the most brittle of instances.

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INTRODUCTION

Everyday experience indicates that strength, or stress at fracture, does not vary with size - this is reflected in the reporting of ultimate stress values for materials in handbooks. However, when size is reduced below that normally encountered in uniaxial tension tests, increases in strength can occur. Such *strength size effects* are especially prevalent in brittle materials. For example, in Griffith's classical paper [1], the breaking stress of glass fibers increases by 70% on reducing their diameter by an order of magnitude, and increases by an additional factor of 5 or so on reducing diameter by another order of magnitude. Clearly then, strength size effects can be considerable when they do indeed occur, and they need to be taken into account in any theory attempting to predict fracture in such cases.

Turning to the fracture of cracked specimens composed of brittle materials, probably the most accepted theory for treating these configurations is linear elastic fracture mechanics (LEFM). Implicit in LEFM, in the choice of the stress intensity factor as the parameter governing fracture, is a strength size prediction. To see this consider, by way of illustration, the scaled pair of single-edge-cracked specimens sketched in Fig. 1. Herein Specimen 1 is a strip of indefinite length yet finite width W, weakened by a crack of length a, and subjected to a remote uniform stress σ_1; while Specimen 2 is also of indefinite length but has width λW, crack length λa, and applied stress σ_2. Thus λ is the in-plane scale factor and we ignore out-of-plane effects for the present. For Specimen 1 at fracture, LEFM has

$$K_I = \sigma_1^* \sqrt{\pi a}\ f(a/W) = K_c,$$

where K_I is the stress intensity factor in Mode I, f(a/W) is a finite width correction factor, K_c is the material fracture toughness, and the asterisk atop σ_1 serves to distinguish it as being the applied stress at fracture. Similiarly

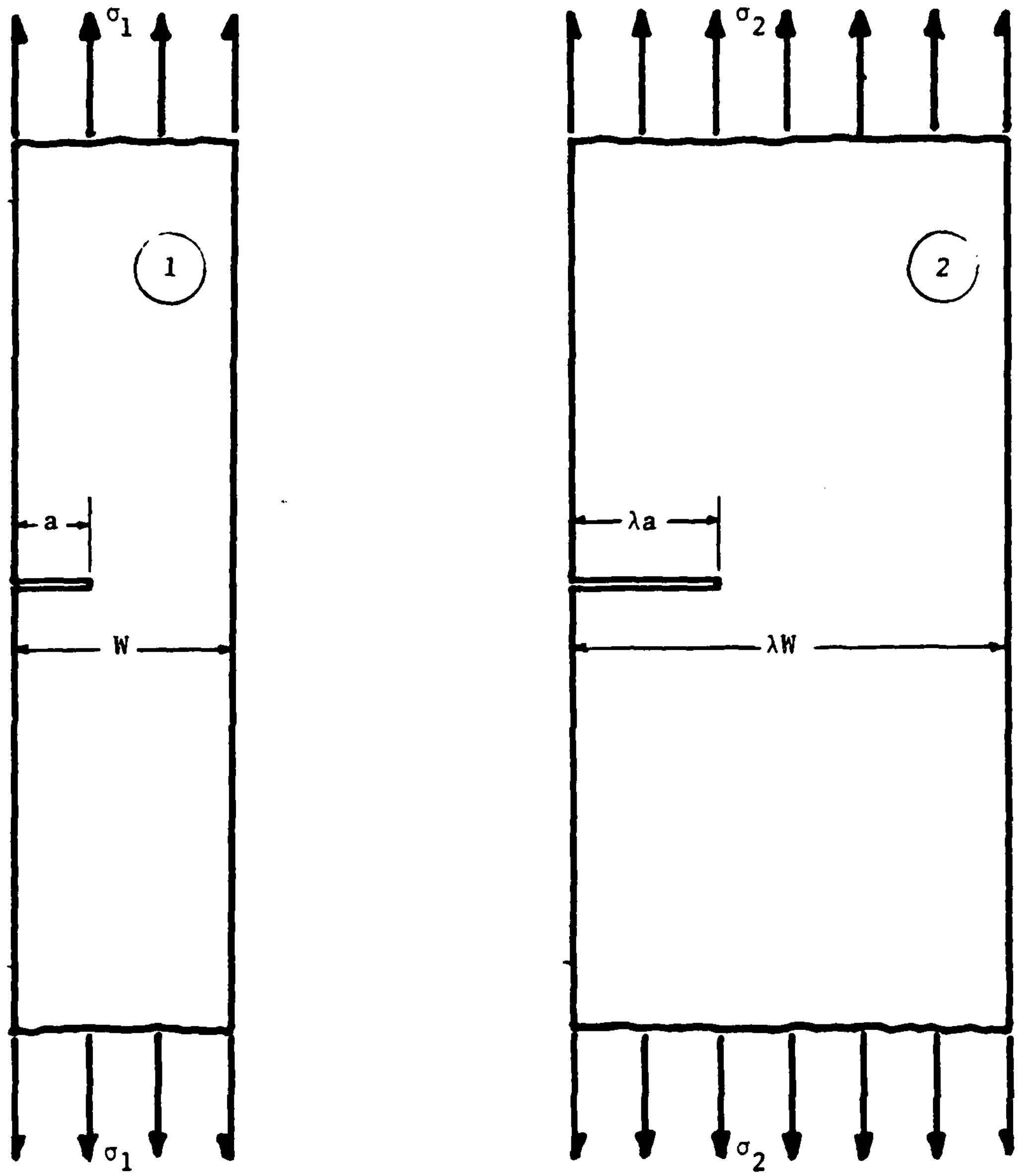

Fig. 1. Scaled pair of single-edge-cracked specimens

for Specimen 2, provided it is comprised of the same brittle material, at fracture

$$K_I = \sigma_2^* \sqrt{\pi\lambda a}\ f(\lambda a/\lambda W) = K_c,$$

whence

$$\sigma_1^*/\sigma_2^* = \sqrt{\lambda}. \qquad (1)$$

The strength size effect prediction of (1), sometimes termed the *geometric size effect* of LEFM, is certainly in agreement with general trends, *viz.*, strength increasing with decreasing size. The question that arises, though, is *how well* it in fact agrees with physical actuality.

The question is important on two counts. First, the ability to predict when changes are limited to scale alone is quite conceivably the easiest of tests a theory can face, and therefore virtually an essential prerequisite to satisfactory performance in more complex contexts. Second, appreciable changes in size are encountered in engineering. For instance, at the Government Products Division of Pratt and Whitney Aircraft Corporation, tests on 6-inch wide panels have been used to infer what is happening at the necks in "fir trees" which hold blades in high pressure turbine disks, and which can be as small as 1/16 of an inch in width. While there is more than just a size change entailed in this specific application, the scaling factor involved is of the order of 100. In sum then, accurately accounting for scaling is a basic capability for linear elastic fracture mechanics to possess, and one which is required in practice.

To answer the question we have reviewed the open literature, drawing on data furnished in papers in a variety of journals, in ASTM Special Technical Publications and proceedings of other conferences related to fracture mechanics, and occasionally in reports. In performing this search we have been fortunate to be directed to a number of references that provide some experimental support for (1), all of which we of course include here. We have attempted to be as comprehensive as we can in supplementing these references, but nonetheless are confident that there remain still further references which we have not, to date, found - we would appreciate readers drawing our attention to any such oversights and be happy to include them in an updated version of the present study. We do not anticipate, though, that the amount of pertinent data outstanding is sufficient to significantly alter the overall assessment made on the basis of the extensive survey summarized here.

In answering the question we have also placed the main emphasis on physical data featuring brittle response, since such phenomena are in greatest accord with the assumptions in linear elasticity and consequently can be expected to agree with LEFM best. We have, in addition, tried to isolate the issue as much as possible, by selecting data as close to perfect scaling as can be drawn from a given source and by taking strength estimates from load data wherever we were able, thereby avoiding practically all analysis. And in processing the data we have made an effort to apply reasonable data reduction procedures in a consistent manner. In this way it is hoped to focus simply on what the physical data has to say about the applicability of the LEFM strength size prediction.

In what follows we begin by describing the ground rules for including data relevant to the issue and how such data are then classified. Next we provide the results in summary form (greater detail can be found in the Appendix), and discuss how well theory and practice agree. Finally we offer some concluding remarks on the consequences of the comparison.

SURVEY GUIDELINES

In this section we start by defining strength. We then place limits on any deviations from scaling, including bounds on thickness effects. With this last in place we can distinguish between brittle versus ductile behavior. In all, the intent here is to furnish a reasonable set of rules which can be systematically applied to filter out data not enabling a fair appraisal of the LEFM size prediction as a result of not conforming to underlying assumptions: we do this either by excluding such data altogether or by separating it into other classes wherein good agreement with fracture mechanics is not necessarily expected.

We define *strength* as the nominal, elastic, net section stress, under monotonic quasi-static loading, at the onset of Mode I crack growth (or further crack growth in the case of flaws that have been previously grown by cyclic loading). With truly brittle response this stress level is usually equal to that required for total fracture; with less brittle behavior it varies somewhat, loads at a 5% secant offset in the load-displacement record being generally preferred but others, such as at "pop-in", being taken when the 5% offset stress is not available. We choose the stress at this point rather than at its maximum because response is likely to be more linear elastic. In the event, however, that only maximum load levels are provided in a given source, we include such data but note the relaxation of our definition. We do this in the interests of admitting as much data as possible that bear on the issue at hand. The attendant hierachy of preferred sorts of information for deducing strengths from then is: load and geometry, including final pre-crack length, at the onset of crack growth; gross stress then net section stress at the same point; fracture toughness (or stress intensity factor) at crack initiation; resistance curves with discernible proportional small crack growth; followed by the same first four quantities, in order, at maximum load. Only the highest available information type in this ranking is used. No attempts are made to infer strengths from load-displacement products, or their equivalents, unless load or stress alone can be determined.

Preliminary to prescribing limits on departures from perfect scaling, we need to define what we mean by a *crack* and describe the range of acceptable *environments*. Here, with respect to a crack, our first choice is a specimen with a fatigue pre-crack. Again, however, with a view to including all pertinent data, we admit notches whose acuity is such that their local root radius of of curvature, r, is an order of magnitude less than their total notch depth or crack length, a, *i.e. notch acuity* must satisfy

$$r/a \leq 1/10. \tag{2}$$

Too, we record whenever the situation is not our ideal of pre-crack and compare the two classes of "cracks". Regarding environments, with a view to maintaining as much control as possible, we do not allow data for any that are more corrosive than air. On the other hand some variations in temperature are countenanced, since low temperature response can be quite embrittled and accordingly in good agreement with LEFM assumptions. When comparing scaled specimens at other than room temperature, we allow differences in temperature between sizes of up to 5°K, with the *proviso* that the specimen having the higher stress-intensity-factor value at failure not be at a higher temperature. This stipulation is so as to avoid mixing temperature transition behavior with size effects.

Turning to *scaling*, our first requirement is that specimen type be identical, *e.g.* a compact tension specimen of one size can only be compared with another compact tension specimen of a different size. No exceptions to this restriction are permitted. Our second requirement is that, if cracks are in fact sharp notches, the notch acuity should scale (r/a constant) or at least r should remain constant. Data where r decreases with increasing scale factor, λ, are excluded. Our third requirement is that the crack length, a, to width ratio, W, scale to within 10%. That is

$$0.9 \leq (a_1/W_1)/(a_2/W_2) \leq 1.1 \tag{3}$$

where subscripts refer to specimen. The only exception to (3) entertained is when a/W for a test piece of one size falls between two a/W for a test piece of a second size and the latter are within ± 20% of the former; then we interpolate between the pair of a/W for the second size. Our fourth requirement concerns the other in-plane dimension of a specimen - length, span, *etc*. This should scale sufficiently so that a stress intensify factor calculation for the different sizes remains unaffected by any changes in its value relative to W. Typically this just means that specimens whould preserve a length which is as long as, or longer than, W. And our fifth requirement in essence is that thickness effects are to be excluded. One means of ensuring this is to have the thickness, B, scale in concert with λ; then whatever combination of plane stress and plane strain states is present in one size is conserved in the next. Alternatively test pieces can maintain a state of plane stress or plane strain exclusively by having a fixed B which is relatively small or large, respectively. To this end we define, in a global sense, *plane stress* as being when

$$B/a \leq 0.1, \tag{4}$$

while *plane strain* is taken as being when

$$B/W \geq 0.5,\ B/a \geq 1.0. \tag{5}$$

The plane stress range is so that the thickness is an order of magnitude less than the smallest of the in-plane dimensions while the plane strain range is motivated by standard specimens in plane strain fracture toughness testing. Clearly, for a fixed B as a, W vary, (4), (5) may exclude quite a number of configurations. To reduce this possibility somewhat we relax (4), (5) to $B/a \leq 0.3$ and $B/W = 0.45$, $B/a \geq 0.9$, respectively. . As previously, we separate data that is only admitted by virtue of this relaxation. For convenience we classify specimens with scaled B as plane stress or plane strain depending on which of (4) or (5) they are closest to.

We now consider the crux of the applicability of linear elastic fracture mechanics - namely how *brittle* the response is. Here by brittle we mean in an engineering sense of limited plastic flow rather than from a materials viewpoint of relating microstructural fracture mechanisms. As a result we need to estimate the extent of the yield region induced at failure. *In lieu* of anything obviously superior, we take the classical measures of yield region radius, r_Y, to this effect.

These are, for plane stress

$$r_Y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_Y}\right)^2, \tag{6}$$

and for plane strain (from Irwin [2])†

$$r_Y = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_Y}\right)^2. \tag{7}$$

In (6), (7), K_I is the Mode I stress intensify factor evaluated at the onset of crack growth, σ_Y the uniaxial yield stress. The factor of 3 difference between (6), (7) is the real reason for our earlier concern to distinguish between plane stress and plane strain. Once r_Y is calculated with the appropriate equation, we regard the response as being brittle provided r_Y is less than 2% of the total crack length, irrespective of whether the specimen in question is in a state of plane stress or plane strain. That is, the *brittle regime* requires that

$$r_Y/a < 0.02. \tag{8}$$

This is essentially the same limit as is prescribed in standards for fracture toughness testing. For specimens with more extensive plastic flow than that permitted in (8) yet short of gross yielding, we term the response brittle-ductile if r_Y is less than 5% of the total crack length, *viz.*, the *brittle-ductile regime* has

$$0.02 \leq r_Y/a < 0.05. \tag{9}$$

This transition class is regarded as one it would seem natural for the practicing engineer to attempt to extend the applicability of LEFM into, given satisfactory performance in the brittle regime. This view is supported to a degree by the large number of articles in the literature that report fracture toughness values in the brittle-ductile regime, implying some sort of use of fracture mechanics is contemplated therein. All other responses are taken as being ductile, *i.e.* the *ductile regime* occurs when

†For a recent review of the ability of (6), (7) to be representative of yield region extent see [3], which indicates that, given the wide variety of configurations encountered in testing, they do perform remarkably well.

$$r_Y/a \geq 0.05. \qquad (10)$$

We include data from specimens in this range simply for completeness.

In applying all of the foregoing, considerable effort is made to avoid comparing apples with oranges. For example, if we have two fatigue-cracked compact tension specimens comprised of identical materials, each having a counterpart which is perfectly scaled by a common factor λ yet having different a/W, we keep the ratios of the strengths, σ_1^*/σ_2^*, apart. Similarly we do not compare specimens in the brittle regime with those in the brittle-ductile. On the other hand, since the choice of 0.05 in (9), (10) is somewhat arbitrary, we let it shift to as high as 0.08 for a last size in a set that is otherwise brittle-ductile, and conversely to drop to 0.04 for one size in a ductile set. Typically too, we try to err on the side of classifying response as more ductile than it is rather than less. Within these separate classes, for two different sizes we form all possible quotients of the strengths for the small specimen divided by those for the large, σ_1^*/σ_2^*, note the range of these ratios and calculate their mean. In this fashion we hope to, if anything, overestimate the extent of scatter in the data by using extremums, yet reduce the effects of scatter by using means. We also take down the total number of tests involved in producing a mean strength ratio.

Clearly the choice of the above procedures is not unique, merely sensible. Other reasonable approaches certainly exist. A number of these may well fall within the concessions made on our most stringent requirements. Thus by monitoring any differences between the strength size dependence of data complying with our preferred constraints and that for data only admitted as a result of a relaxation, we should be able to assess what, if any, effects such alternative treatments would have. The expectation is that for these and other rational data reduction schemes, while they would give rise to differences in detail for individual comparisons, they would nonetheless yield the same overall appreciation of how well the LEFM size prediction works if consistently applied to *all* the physical evidence.

We apply our set of rules to every one of the references with strength size effect data encountered in our search. Even on the basis of our more generous restrictions, this excludes the data from some because of not scaling properly, not isolating thickness effects, not having sufficient notch sharpness, *etc.* Too, there is quite a bit of duplication in the reporting of data, *e.g.* the well-known appraisal of fracture mechanics by Weiss and Yukawa in STP 381, [4], actually discusses data first furnished elsewhere. In these cases we only cite the original source.† A listing of all of the around 300 references checked may be found in the bibliography [5]; the 100 odd references determined as independent sources of admissible data are listed alphabetically by author here [6-136]. We next look to examine the outcome of comparing the data from these sources with the strength size prediction of fracture mechanics.

RESULTS AND DISCUSSION

In this section we begin by reviewing the ramifications of our varying admission standards for data and grouping it accordingly. We then display plots of strength ratio data versus scale factor, together with the LEFM prediction of the same. Finally we introduce measures to quantify agreement and discuss their implications.

Considering the various relaxations in requirements we observe that, for the most part, they are of no consequence because data fall within our most stringent requirements and sharpest classifications. Regarding each of those instances in turn where there are exceptions, we first remark that strength data based on maximum loads/stresses tend to exhibit less of a size effect than that associated with strengths at the onset of crack growth. However, where size effects are distinct for the two strength types, the maximum criterion usually involves large stresses with attendant extensive yielding. Consequently nearly all such data

†If the original source is an in-house report with substantially the same authorship as a version in the more open literature, we give the latter because of its greater accessibility. On occasion, when we have not managed to obtain a copy of an original source, we list it together with the reference reporting its data.

falls in the ductile regime and we admit them as such. Second, *re* notch acuity, we find there is some but little effect due to notch sharpness provided (2) is met; typically notches with less acuity display less in the way of size effects but there is no clear distinction in the range allowed by (2). Hence we use data from both notches and fatigue pre-cracks but, if in fact it is not from the latter, note the notch acuity in the detailed tables in the Appendix. Third, concerning the extended ranges of a/W similarity and B/a, B/W values for avoiding thickness effects, we find no significant differences in strength size effects between data fulfilling our strictest requirements and that only complying with our more generous limits. As a result we combine the two types in what follows. In contrast, there can be variations in strength size effects for sets of scaled specimens that have distinct a/W but are otherwise the same. Consequently we continue to segregate such data in computing mean σ_1^*/σ_2^* and note when this occurs in the tables in the Appendix. These tables give associated sources and are broadly classified as to material type; the bulk of the data presented in the remainder of this section are for steels and aluminum alloys.

Distinguished as to plane strain or plane stress and by brittle, brittle-ductile, ductile regime, Fig. 2 shows test data for strength ratios, σ_1^*/σ_2^*, plotted versus scale factor, λ. The size of the dots reflects the number of tests involved Also shown is the LEFM prediction (1). Agreement is not great. Even in a best fit sense, the LEFM prediction is generally astray. More precisely, using least squares weighted by the number of tests involved to determine k in

$$\sigma_1^*/\sigma_2^* = \lambda^k, \qquad (11)$$

leads to: for plane strain brittle, brittle-ductile, ductile, k = 0.37, 0.35, 0.22 respectively while for the corresponding regimes for plane stress k = 0.49, 0.41, 0.22 (*cf.* 0.50). The only category in good agreement with LEFM is plane stress brittle, although naturally one would not expect matching of LEFM and the data in the

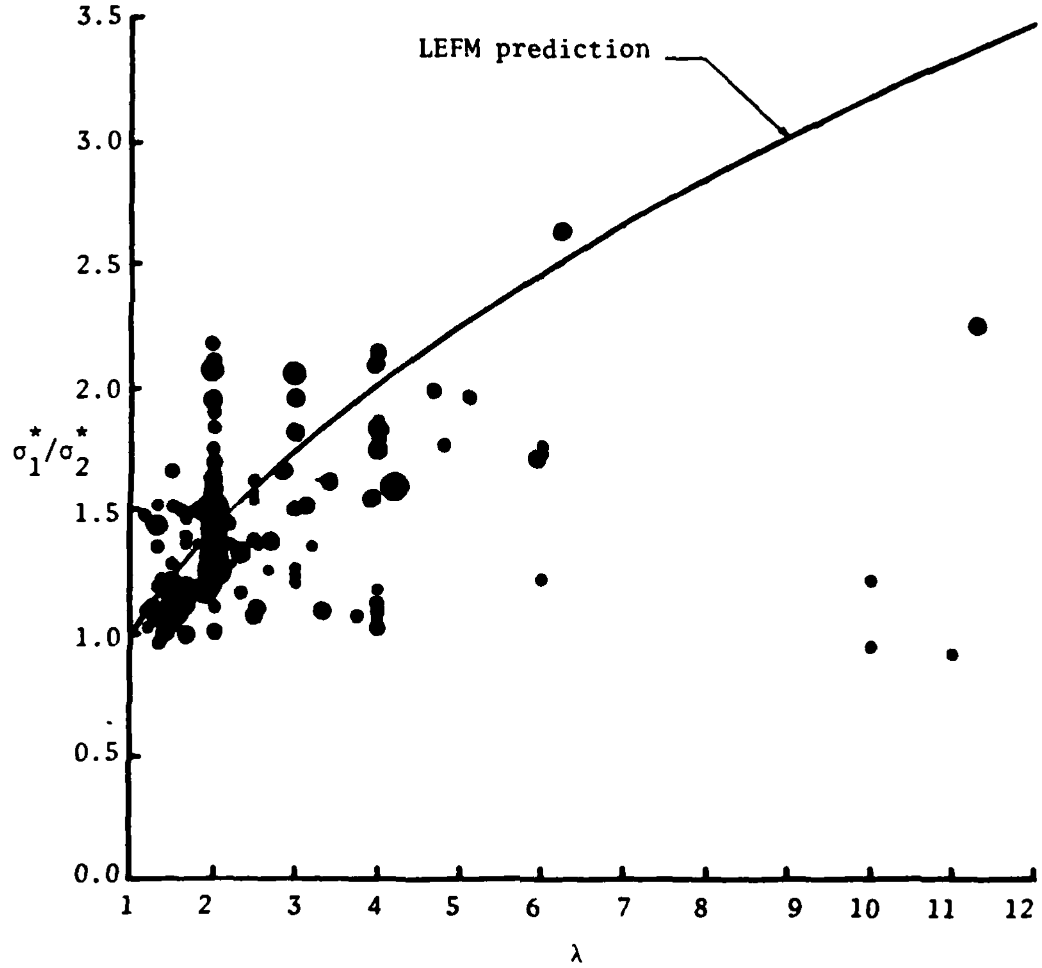

Fig. 2. Comparison of test data with LEFM predictions of strength size dependence:

(a) plane strain, brittle

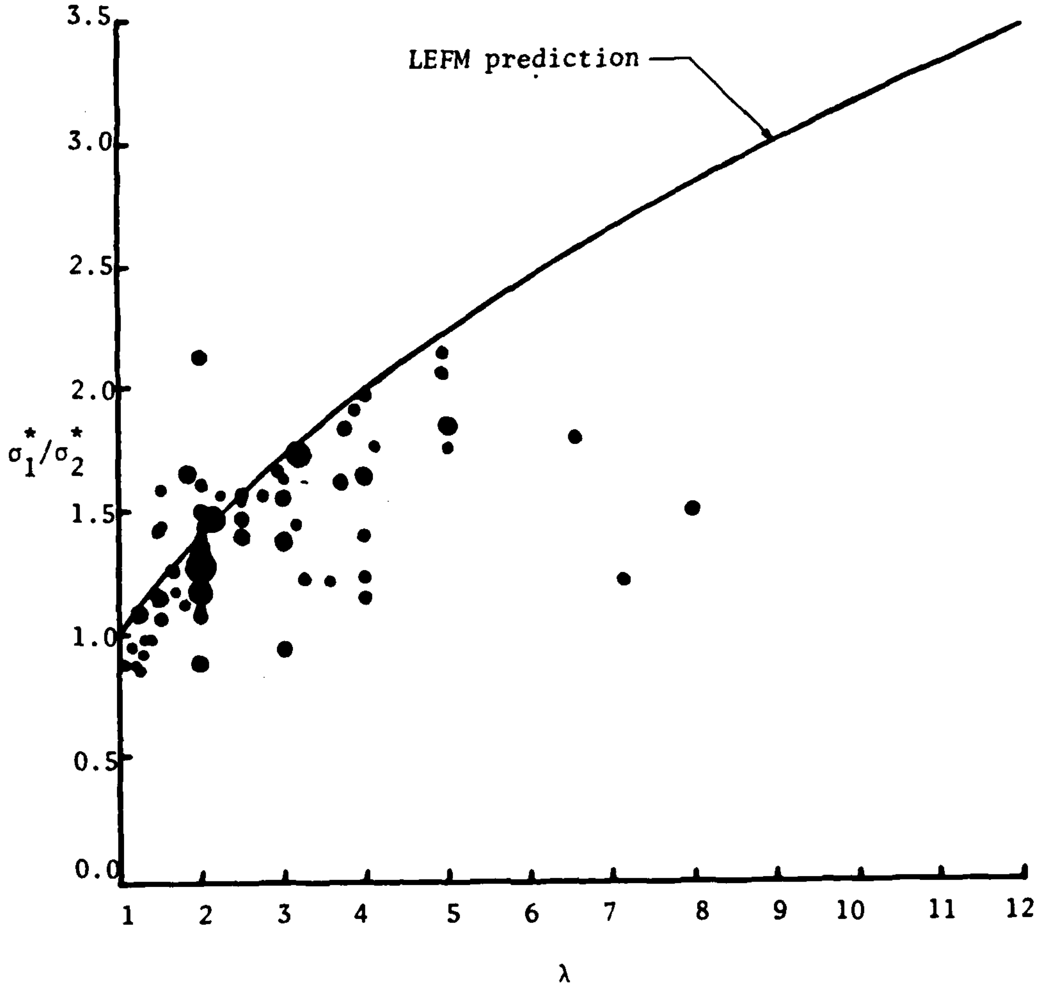

Fig. 2. Comparison of test data with LEFM prediction of strength size dependence: (b) plane strain, brittle-ductile

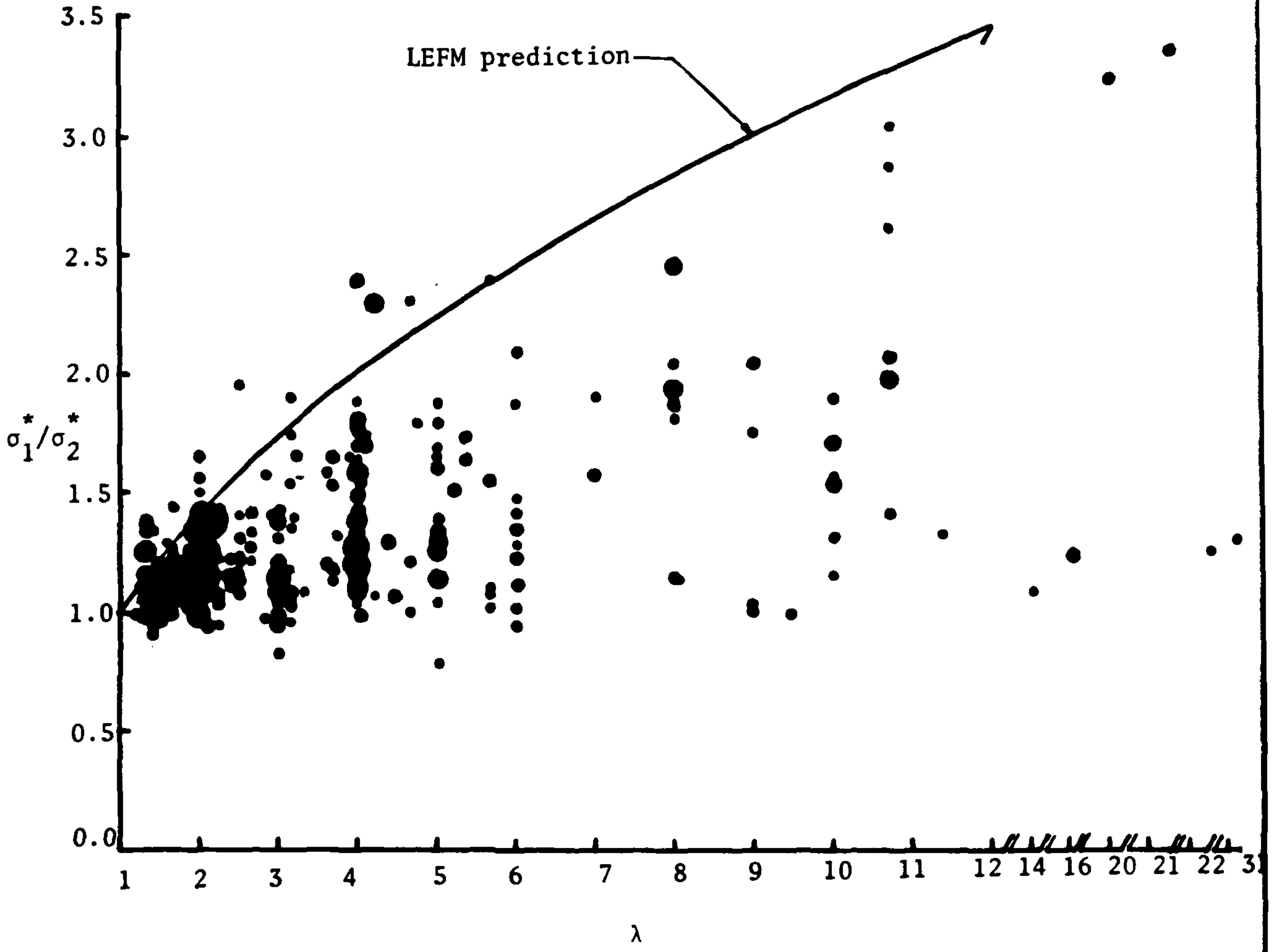

Fig. 2. Comparison of test data with LEFM prediction of strength size dependence:

(c) plane strain, ductile

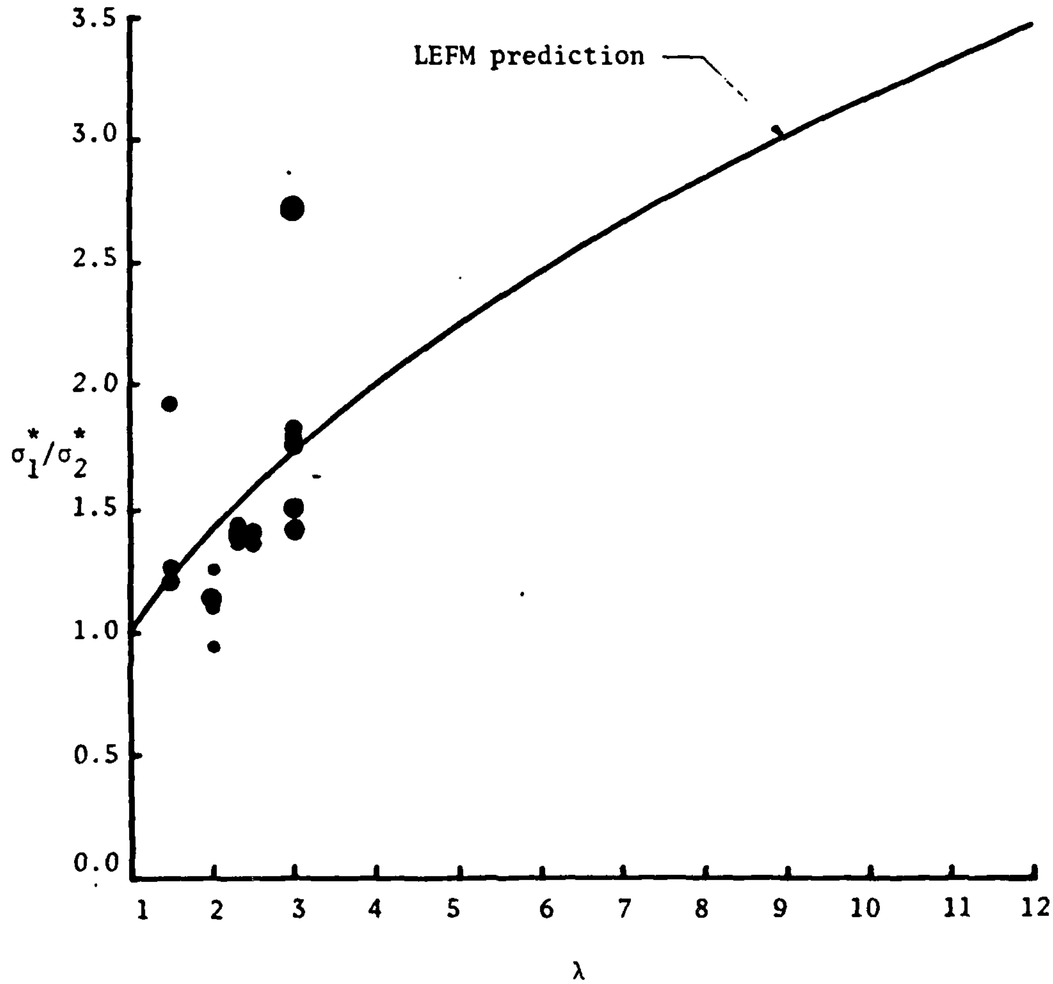

Fig. 2. Comparison of test data with LEFM prediction of strength size dependence: (d) plane stress, brittle

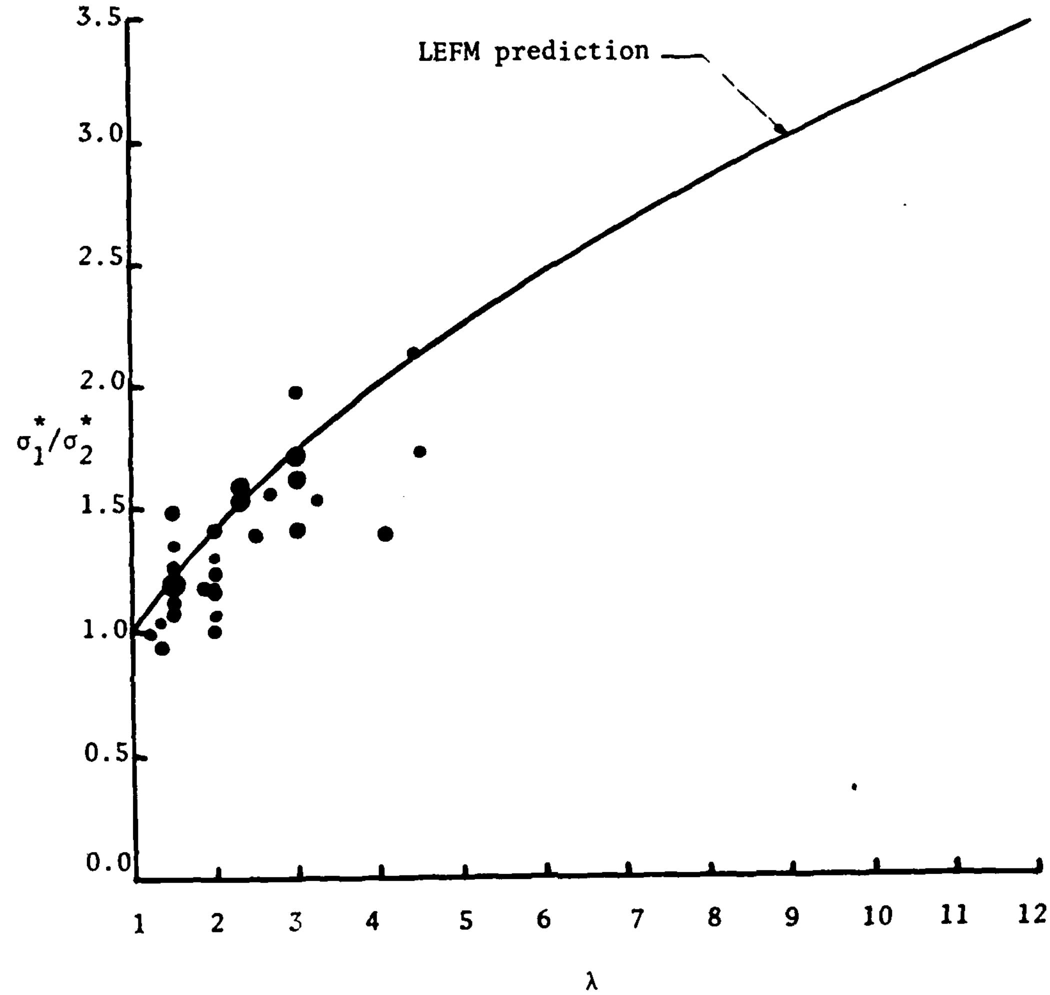

Fig. 2. Comparison of test data with LEFM prediction of strength size dependence: (e) plane stress, brittle-ductile

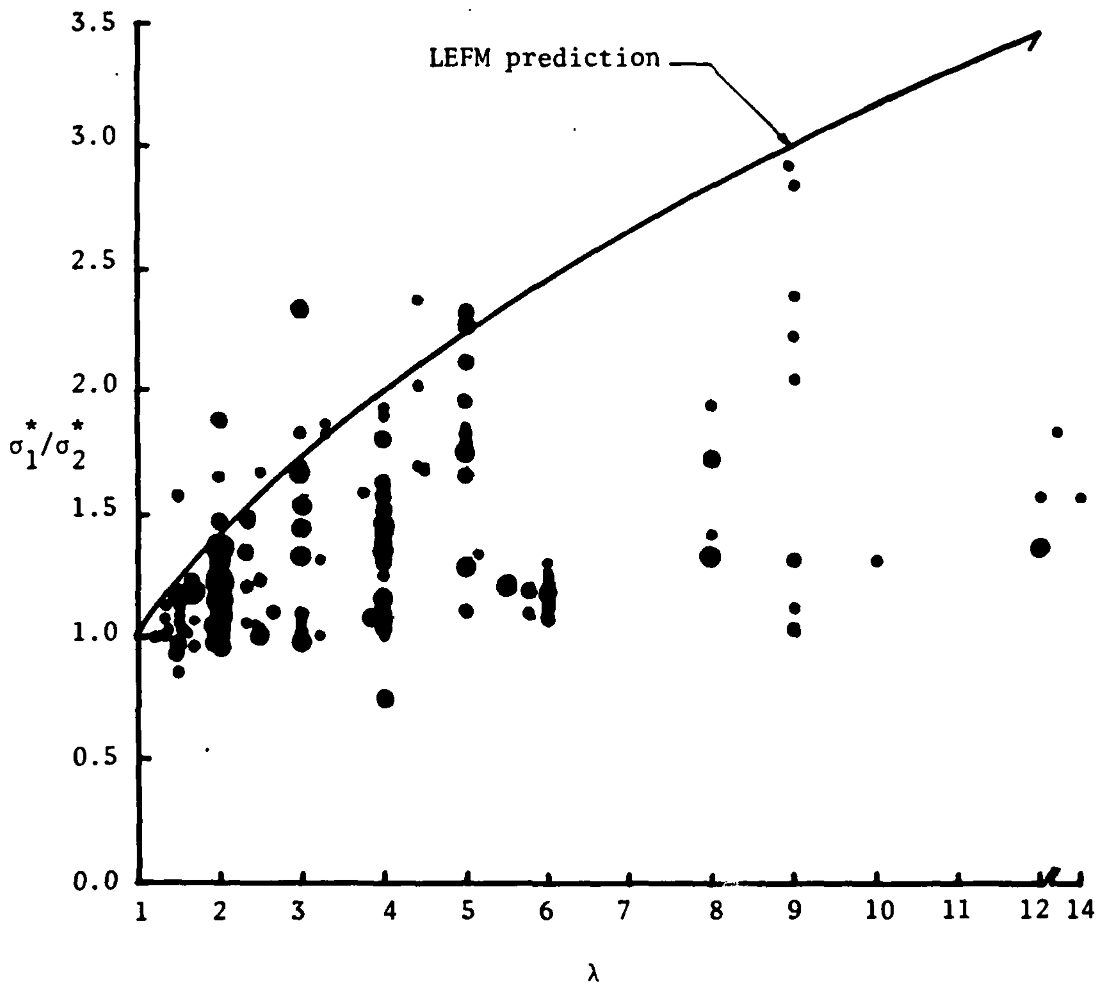

Fig. 2. Comparison of test data with LEFM prediction of strength size dependence

(f) plane stress, ductile

ductile regime. The real point is that no one curve fits any of the data distributions in any of the plots in Fig. 2 well. And this shortcoming cannot simply be dismissed as due to scatter because a significant proportion of the values plotted represent means themselves and, moreover, in over half of the instances where ranges can be calculated from the extreme values of σ_1^*/σ_2^*, these intervals do not even intersect the LEFM prediction irrespective of how brittle response is. We next consider means of quantifying the discrepancies between theory and actuality apparent in Fig. 2.

One way of gauging the effectiveness of the LEFM strength size prediction is to ask how often it does indeed predict the strength of one specimen given the strength of another geometrically similar one. Thus we regard LEFM as providing a *good prediction* if the data are within $\pm$ 5% of (1), a *useful prediction* if within $\pm$ 10%. Table 1 summarizes the percentages of the data that fall within either of these two ranges, the percentages being arranged under the same separate classes as Fig. 2.[†] In the light of Fig. 2, the unsatisfactory percentages in Table 1 are not unexpected. As before, the ductile regime is worst, but of course could be reasonably discounted in evaluating LEFM's strength size prediction. Somewhat suprisingly, the brittle-ductile regime has higher percentages in some sort of agreement with LEFM than the brittle regime. Nonetheless, neither is very satisfactory with LEFM not being within useful agreement with about half of the data. Furthermore, a lot of what agreement there is comes from limited changes in scale ($\lambda \leq 2$), when there is really very little to predict.

To expand on this last statement, Fig. 3 shows how the percentages with useful and good agreement vary with scale factor. The exact histogram classes in Fig. 3 include their upper marks, and the data are drawn from the plane strain brittle regime. Other data exhibit similar behavior. Evident in Fig. 3 is the falling off

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[†]The percentages are weighted by the number of tests involved - no real changes occur if this not done.

Table 1. LEFM strength prediction

Thickness classification	Material response regime	No. of tests involved	% within $\pm$ 10% of LEFM	% within $\pm$ 5% of LEFM
Plane strain	Brittle	699	37%	18%
	Brittle-ductile	281	44%	23%
	Ductile	1331	15%	5.5%
Plane stress	Brittle	103	44%	26%
	Brittle-ductile	119	55%	40%
	Ductile	663	20%	8.9%

as λ increases of the agreement of the LEFM size prediction with the data. Eventually ($\lambda > 7$), *no* data are within $\pm$ 10% of (1). Although there are fewer tests for these higher scale factors, there would seem to be sufficient to show some agreement if indeed it were to be present (the numbers of tests involved within each class, and upon which percentages are based, are given in Fig. 3 in parentheses).

Motivated by a desire to reflect the trend apparent in Fig. 3, we introduce the *size effect*, s, as being the deviation from strength size independence. Thus

$$s = \sigma^*_1/\sigma^*_2 - 1 = \sqrt{\lambda} - 1, \qquad (12)$$

according to LEFM. Examining whether the data is in useful ($\pm$ 10%) or good ($\pm$ 5%) agreement with s of (12) tends to remove the almost automatic agreement for low λ inherent in our check on strength predictions. The percentages of the data complying to the two degrees with LEFM's *size* prediction are presented in Table 2, simply grouped under brittle, brittle-ductile and ductile since this is the true key to applicability, rather than plane stress versus plane strain. Without the easy acceptance limits for small changes of scale of the measures in Table 1, LEFM is shown to be quite ineffective in its ability to track size dependence in Table 2.

At this point it is natural to ask if perhaps satisfactory performance of linear elastic fracture mechanics requires a more restrictive set of circumstances than even the most applicable admitted here. Two options along these lines are to examine whether or not data stem from valid K_{IC} testing, and to attempt to separate out still more brittle behavior.

Concerning the first, it is not trivial to ascertain whether or not data is from valid K_{IC} tests according to current ASTM standards [137]. The reason for this is that normally insufficient information is given in articles - we did not come across a single paper which gave all the information required to check all of the E399 requirements in [137]. As a result, unless we could detect an aspect which infringed present standards, we simply took it on contributors own statements as

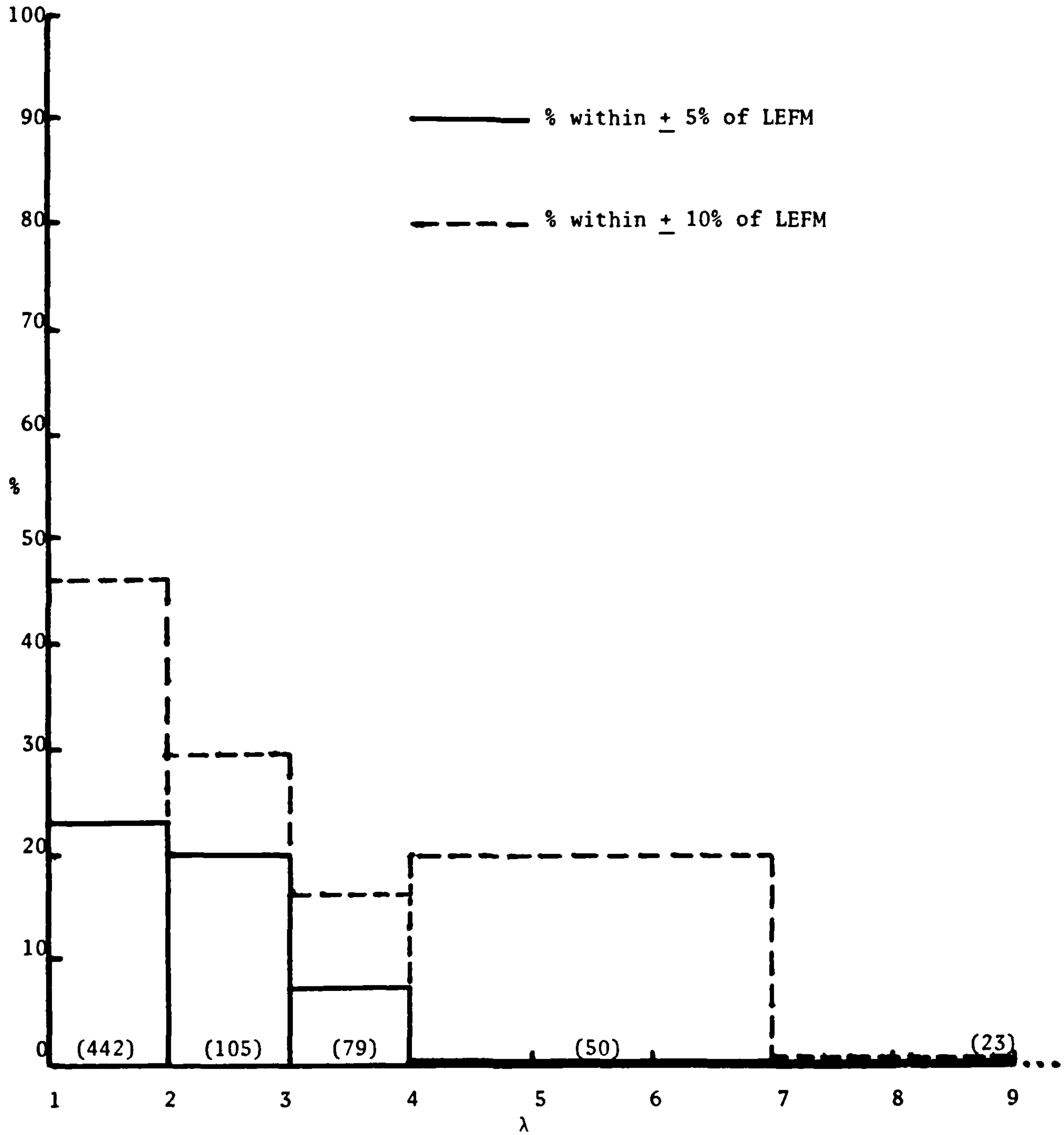

Fig. 3. Strength prediction agreement with varying scale factor for brittle response

Table 2. LEFM size prediction

Material response regime	No. of tests involved	% within $\pm$ 10% of LEFM	% within $\pm$ 5% of LEFM
Brittle	802	12%	4.2%
Brittle-ductile	400	18%	9.3%
Ductile	1994	4.4%	1.8%

to whether their data is valid K_{IC} or not. The agreement in terms of size effect prediction so as to avoid undue weight to low-λ values, is presented in the top half of Table 3. It would be difficult to argue that there is any significant improvement offered by the valid K_{IC} data over other plane strain brittle data.

Regarding the second, to see if there is a trend towards greater agreement with increasingly brittle response, we merely divide the same data set - plane strain, brittle - into two. We do this by defining a very brittle regime when $0 \leq r_Y/a < 0.01$ and a quasi-brittle regime when $0.01 \leq r_Y/a < 0.02$. Some of the data could not be segregated in this way. The agreement for the remainder, in terms of size effect prediction, is presented in the bottom half of Table 3. Again, it would be hard to establish there being any significant improvement.

In sum then, the physical data is not in satisfactory agreement with the strength size effects prediction of linear elastic fracture mechanics, especially when there are appreciable changes in scale, and this unsatisfactory situation appears to persist even when considerable effort is expended to conform with the assumptions underpinning LEFM.

CONCLUDING REMARKS

The prediction of strength size effects in fracture mechanics concurs with trends in physical data but is so naively simple as to be manifestly incomplete. Accordingly it can lead to predictions that are inaccurate to the point of not being acceptable in engineering. Moreover, such errors are typically not conservative when testing a specimen larger than the size of the intended application. And this situation can certainly arise in practice (recall the turbine disk example in the Introduction).

In practice, then, it is preferable, if not necessary, to test on the same size scale as the application. In the event of this being impractical, the following

Table 3. LEFM size prediction for different types of brittle response

Classification	No. of tests involved	% within $\pm$ 10% of LEFM	% within $\pm$ 5% of LEFM
Valid K_{IC}	190	11%	4.7%
Not K_{IC}	509	9.6%	1.4%
Very brittle	158	10%	7.6%
Quasi-brittle	349	9.5%	4.6%

strategy might be adopted. For the most part, the size effects predicted by LEFM are in excess of those for the actual data. Hence when testing small and applying big, LEFM can be used to estimate the reduction in strength and usually will do so conservatively. On the other hand, when testing big and applying small, the LEFM prediction can greatly exceed the strength increases in fact realized. Here, though, it would appear that strength seldom decreases with decreasing size. Consequently, on nondimensionalizing LEFM by dividing the stress intensity factor by the specimen width at the crack to furnish a size independent parameter, allowances for size effects will generally be conservative.

A caution on the use of the above is in order. There is really no physical reasoning underlying the scheme; it is merely based on observation of the data. And this data is not always confined within LEFM's prediction of size effects and size independence so that there is physical evidence of the strategy being nonconservative.[†] Some judgement is therefore required in implementing this essentially empirical approach.

On a more fundamental front, there is reason to be concerned about the very basis of fracture mechanics. These concerns arise because there exist several hundred test results for appropriately brittle behavior *not* agreeing with the LEFM prediction of strength size effects. Every one of these represents data establishing a variation in fracture toughness with size. It follows that fracture toughness is demonstrably *not* a material property. Thus the use of the stress intensity factor as the parameter in and of itself controlling brittle fracture needs serious examination.

Acknowledgements - We are pleased to acknowledge the valuable benefit of discussions during the course of this work with J.H. Griffin of the Department of Mechanical Engineering at Carnegie-Mellon University and with D.A. Glasgow of the Air Force Office of Scientific Research (AFOSR). The financial support of AFOSR is also appreciated.

[†] There are a significant number of points above the LEFM prediction in Fig. 2 with ranges that do not extend down sufficiently to intersect the LEFM prediction, and conversely there are points below size independence ($\sigma_1^*/\sigma_2^* = 1$) with ranges not intersecting the same, although there are no ranges not including $\sigma_1^*/\sigma_2^* = 1$ in the brittle regime.

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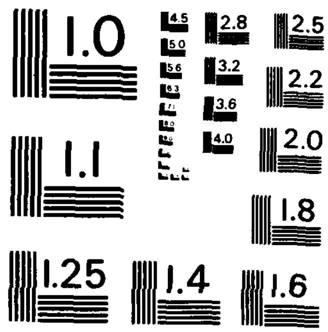

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APPENDIX

Here we tabulate all sources used together with a brief description of the testing and the actual data taken as well as its classification. Within the tables, sources are arranged alphabetically by author surname. For the test description we employ the following abbreviations for specimen type: CCP...center-cracked plate, CTS...compact tension specimen, CVN...Charpy V-notch, RCT...round compact tension, SEN...single-edge notch, TNF...thumbnail flaw, VNC...V-notched cylinder, WOL...wedge opening loading, 3PB...three-point bend, and 4PB...four-point bend. After specimen type (s), there is a sequence of quantities in parenthesis which can have as many entries as (n, r/a, T, a/W). The first, n, is always provided and is the number of distinct tests involved in all of the data gleaned from a single source. Thus the total n is somewhat less than the combined number of tests recorded in Table 1, since there some tests are involved in more than one ratio and are therefore counted twice, whereas here each is counted but once. In instances where the precise number of tests could not be determined, n is a greatest lower bound. The second, r/a, is the maximum dimensionless notch radius if in fact notches are employed; no correspondi entry implies fatigue pre-cracks only. The third, T, indicates inclusion of data for temperatures other than room temperature (RT); no T means all data at RT. The fourth, a/W, shows that some of the mean stress ratios are for pairs of scaled specimens whose only distinguishing feature is distinct a/W; no a/W means there is no appreciable variation in relative crack length. Finally by way of explanation, the abbreviations used in the classification are: $p\sigma$...plane stress, $p\varepsilon$...plane strain, b...brittle, b-d...brittle-ductile, and d...ductile. The tables in order present information for steels, aluminum alloys, other metals and non metals.

Table 4. Sources and data for steels

Source and test	Scale factor, λ	Mean stress ratio, σ_1^*/σ_2^*	Classification
Akita [7],	2.10	1.07	pε, d
3PB (4, 1/10)	4.20	1.06	pε, d
Andrews *et al.* [8], RCT (3,T)	2.00	1.25	pε, b-d
Banerjee [10],	8.00	1.34	pσ, d
CTS (4)	12.00	1.38	pσ, d
Batte *et al.* [12],	1.50	1.06	pε, d
SEN (8)	2.00	1.13	pε, d
	3.00	1.11	pε, d
Begley and Landes [13],	2.00	1.06	pε, d
CTS and 3PB (8,T)		1.18	pε, d
Begley and Toolin [14], CTS (2,T)	1.50	1.02	pε, b
Boodberg *et al.* [16],	2.00	1.01	pσ, d
CCP (56, 1/160, T)		1.05	pσ, d
		1.06	pσ, d
		1.06	pσ, d
		1.08	pσ, d
		1.09	pσ, d
		1.10	pσ, d
		1.11	pσ, d
	4.00	1.08	pσ, d
		1.15	pσ, d
		1.15	pσ, d
		1.15	pσ, d
		1.16	pσ, d
		1.17	pσ, d
	6.00	1.08	pσ, d
		1.13	pσ, d
		1.15	pσ, d
		1.15	pσ, d
		1.17	pσ, d
		1.17	pσ, d
		1.18	pσ, d
		1.18	pσ, d
		1.22	pσ, d
		1.23	pσ, d
		1.25	pσ, d
		1.27	pσ, d
	9.00	1.04	pσ, d
		1.31	pσ, d
Brown *et al.* [18],	2.00	1.10	pε, d
VNC (12, 1/36)	4.00	1.15	pε, d
	8.00	1.18	pε, d
	16.00	1.25	pε, d
Brown and Srawley [19],	2.00	1.36	pε, b
4PB and SEN (57, a/w)		1.36	pε, b
		1.38	pε, b
		1.39	pε, b
		1.45	pε, b
		1.48	pε, b
		1.51	pε, b
		1.54	pε, b-d
		1.63	pε, b
	3.00	1.82	pε, b
		1.96	pε, b
Chell and Davidson [22],	1.67	1.28	pε, d
SEN (10, a/w)		1.45	pε, d
	1.88	1.21	pε, d
	3.13	1.34	pε, d
		1.73	pε, d
		1.91	pε, d
Chell and Gates [23],	1.88	1.25	pε, d
SEN (10, a/w)		1.34	pε, d
	3.13	1.17	pε, d
		1.54	pε, d
Chell and Spink [24], 3PB (3)	4.00	1.48	pε, d
Christian *et al.* [25],	2.00	1.18	pσ, b-d
CCP (14, 1/500, T)		1.21	pσ, d
		1.23	pσ, d
		1.31	pσ, d
	3.25	1.48	pσ, d
		1.53	pσ, b-d
	4.50	1.69	pσ, d
		1.72	pσ, b-d
	9.00	2.05	pσ, d
		2.24	pσ, d
Clark *et al.* [27],	2.00	1.34	pε, d
CTS (29, T)		1.36	pε, d
		1.37	pε, d
	4.00	1.43	pε, d

Table 4 continued

Source and test	Scale factor, λ	Mean stress ratio σ_1^*/σ_2^*	Classification
DeSisto *et al.* [30], VNC (9, 1/23)	1.41	0.95	pε, d
	2.23	1.01	pε, d
	3.16	1.09	pε, d
	4.46	1.05	pε, d
	10.00	1.16	pε, d
	14.06	1.11	pε, d
	22.31	1.28	pε, d
	31.56	1.30	pε, d
Elsender *et al.* [31], as reported in Chell and Gates [23], SEN (10, a/w)	1.50	1.02	pε, d
	2.00	1.11	pε, d
	3.00	1.20	pε, d
		1.43	pε, d
Ferguson and Sergisson [33], VNC (8)	1.25	1.10	pε, d
	1.47	0.98	pε, d
	1.75	1.11	pε, d
	2.01	1.15	pε, d
	2.51	1.22	pε, d
	3.01	1.37	pε, d
	4.02	1.55	pε, d
Greenburg *et al.* [38], WOL (42,T)	1.33	0.96	pε, b
	1.50	1.66	pε, b
	2.00	0.97	pε, b
		1.19	pε, b
		1.26	pε, b
		1.27	pε, b
		1.27	pε, b
		1.43	pε, b
		1.49	pε, b
		1.50	pε, b
		1.58	pε, b
		1.69	pε, b
	3.00	1.09	pε, b
		2.26	pε, b
Hasofer [41], VNC (89)	1.50	1.06	pε, b-d
Hawthorne and Mager [42], CT (2,T)	4.00	1.13	pε, b
Heyer and McCabe [43], WOL (9)	2.00	0.94	pσ, b
		1.00	b-d
		1.07	b-d
		1.25	b
Huang and Gelles [45], CTS (2,T)	2.00	1.03	pσ, d
Ikeda *et al.* [47], CTS (3,T)	3.00	1.21	pε, b
		1.23	pε, b
		1.25	pε, b
	6.00	1.21	pε, b
		1.72	pε, b
		1.74	pε, b
Jones and Brown [48], 3PB (17)	1.85	1.18	pσ, b-d
	2.20	1.37	pε, b
	3.70	1.62	pε, b-d
	4.07	1.40	pσ, b-d
Kaiser and Hagedorn [50], CTS (4)	2.00	1.21	pε, d
	6.00	1.47	pε, d
Keller and Munz [56], CTS (4)	1.79	1.29	pε, b-d
	3.57	1.21	pε, b-d
	7.14	1.22	pε, b-d
Klier and Weiss [57], VNC (35, 1/44)	1.67	1.06	pε, d
		1.09	pε, d
		1.24	pε, d
	3.67	1.13	pε, d
		1.18	pε, d
		1.52	pε, d
		1.64	pε, d
	5.00	1.04	pε, d
		1.25	pε, d
		1.61	pε, d
		1.79	pε, d
Kobayashi *et al.* [58], CTS (4)	2.00	2.13	pε, b-d
Krasowsky *et al.* [60], 3PB (88, T)	2.00	1.00	pε, d
		1.11	pε, d
		1.18	pε, b-d
		1.39	pε, b-d
		1.40	pε, b
		1.45	pε, b
		1.51	pε, b
	5.00	1.29	pε, d
	7.50	1.57	pε, d

Table 4 continued

Source and test	Scale factor, λ	Mean stress ratio σ_1^*/σ_2^*	Classification
Kuhn [62], CCP (5)	1.50	0.85	pσ, d
	2.00	1.03	pσ, d
Landes and Begley [64], CTS (6,T)	1.50	1.13	pϵ, d
	2.00	1.13	pϵ, d
	4.00	1.59	pϵ, d
	6.00	1.88	pϵ, d
	8.00	1.82	pϵ, d
Logsdon [66], CTS (6,T)	1.33	1.20	pϵ, b
	1.50	0.99	pϵ, b
Logsdon [67], CTS (2)	1.33	1.51	pϵ, b
Logsdon and Begley [68], CTS (9,T)	1.50	1.03	pϵ, b
	2.00	1.18	pϵ, b
		1.65	pϵ, b-d
Lubahn [69], 3PB and VNC (46, 1/12)	1.67	1.07	pϵ, d
		1.09	pϵ, d
		1.27	pϵ, d
	2.00	1.01	pϵ, d
		1.12	pϵ, d
	2.09	1.15	pϵ, d
	2.20	1.11	pϵ, d
	3.67	1.18	pϵ, d
		1.24	pϵ, d
		1.26	pϵ, d
		1.61	pϵ, d
		1.62	pϵ, d
		1.76	pϵ, d
	4.00	1.20	pϵ, d
	4.38	1.29	pϵ, d
	5.00	1.13	pϵ, d
		1.14	pϵ, d
		1.38	pϵ, d
		1.57	pϵ, d
		1.68	pϵ, d
		1.79	pϵ, d
		2.17	pϵ, d
	8.58	1.14	pϵ, d
	8.88	1.94	pϵ, d
	22.32	3.37	pϵ, d
Lubahn [70], 3PB and VNC (24, 1/12)	1.60	1.07	pϵ, d
	2.00	1.14	pϵ, d
	2.40	1.12	pϵ, d
	3.60	1.20	pϵ, d
	4.00	1.29	pϵ, d
	5.20	1.51	pϵ, d
	5.33	1.74	pϵ, d
	8.00	1.94	pϵ, d
	9.00	2.04	pϵ, d
	10.67	1.97	pϵ, d
	21.33	3.36	pϵ, d
Lubahn and Yukawa [71], 3PB (18, 1/13)	2.00	1.13	pϵ, d
		1.14	pϵ, d
	3.27	1.22	pϵ, b-d
	4.00	1.25	pϵ, d
		1.29	pϵ, d
	8.00	1.93	pϵ, d
	9.00	1.77	pϵ, d
	20.00	3.27	pϵ, d
Macdonald [72], CCP and 3PB (14, a/w)	3.00	1.06	pϵ, d
		1.16	pϵ, d
		1.21	pϵ, d
	4.00	2.41	pϵ, d
Markstrom [73], CT (16)	5.00	1.46	pσ, d
		1.74	pσ, d
Markstrom [74], 3PB (14,T)	2.50	1.05	pϵ, b
		1.85	pϵ, b
	4.00	1.01	pϵ, b
		1.04	pϵ, b
		1.17	pϵ, b
	10.00	1.21	pϵ, b
Milne and Worthington [77], 3PB (19 T, a/w)	2.40	1.14	pϵ, d
		1.15	pϵ, d
		1.16	pϵ, d
		1.23	pϵ, d
		1.23	pϵ, d
	6.00	1.38	pϵ, d
		2.11	pϵ, d

Table 4 continued

Source and test	Scale factor, λ	Mean stress ratio σ_1^*/σ_2^*	Classification
Munro and Adams [79],	1.50	1.16	$p\sigma$, d
3PB (56)	2.00	1.38	$p\sigma$, d
	4.00	1.36	$p\sigma$, d
	5.50	1.21	$p\sigma$, d
Murayama *et al.* [82],	1.49	1.44	$p\epsilon$, b-d
3PB and CTS (6,T)	1.52	1.44	$p\epsilon$, d
		1.59	$p\epsilon$, b-d
Neale [83],	2.00	1.29	$p\epsilon$, b-d
CTS (6)	4.00	1.97	$p\epsilon$, b-d
Parker [88],	2.00	1.06	$p\sigma$, d
CCP (9)		1.10	$p\sigma$, d
	4.00	1.09	$p\sigma$, d
	8.00	1.10	$p\sigma$, d
		1.30	$p\sigma$, d
	9.00	1.03	$p\sigma$, d
		1.34	$p\sigma$, d
Pascover *et al.* [89],	1.50	0.96	$p\sigma$, d
CCP (4, a/w)		1.57	$p\sigma$, d
Putatunda and Banerjee [93],	2.00	1.20	$p\sigma$, d
CTS (13)	4.00	1.43	$p\sigma$, d
	8.00	1.73	$p\sigma$, d
Rolfe and Novak [95],	1.50	1.12	$p\sigma$, b-d
3PB (8)		1.22	$p\epsilon$, b
Rolfe and Novak [96],	1.33	1.16	$p\epsilon$, d
4PB (6)	1.50	0.97	$p\epsilon$, d
	1.67	1.24	$p\epsilon$, d
	2.00	1.11	$p\epsilon$, d
Royer *et al.* [97],	2.00	1.41	$p\epsilon$, d
3PB and CTS (9)		1.49	$p\epsilon$, d
	4.00	1.36	$p\epsilon$, d
		1.76	$p\epsilon$, d
		1.89	$p\epsilon$, d
	8.00	1.81	$p\epsilon$, d
Server *et al.* [102],	2.00	1.34	$p\epsilon$, b
3PB (7)	2.54	1.36	$p\epsilon$, b
	5.08	1.96	$p\epsilon$, b
Shabbits *et al.* [103], as reported in Server and Wullaert [101],	2.00	1.21	$p\epsilon$, b
CTS (5)	4.00	1.13	$p\epsilon$, b
	10.00	0.94	$p\epsilon$, b
	11.00	0.92	$p\epsilon$, b
Shannon *et al.* [104],	2.00	1.39	$p\epsilon$, b-d
DEN (36)		1.40	$p\sigma$, d
		1.41	$p\epsilon$, b-d
		1.48	$p\epsilon$, b
		1.50	$p\sigma$, b
		1.62	$p\sigma$, d
Shearin *et al.* [105]	3.00	0.83	$p\epsilon$, d
4PB (5, 1/10)	3.13	0.96	$p\epsilon$, d
	5.00	0.79	$p\epsilon$, d
Shih and Clarke [106], CTS (2)	1.50	1.06	$p\epsilon$, b
Soete [107],	3.00	0.98	$p\sigma$, d
CCP (20, a/w)		0.98	$p\sigma$, d
		1.07	$p\sigma$, d
	9.00	1.13	$p\sigma$, d
Special ASTM Committee,	2.31	1.06	$p\sigma$, d
Third Report [108],		1.20	$p\sigma$, d
CCP (105, 1/1000)		1.34	$p\sigma$, d
		1.37	$p\sigma$, b
		1.39	$p\sigma$, b
		1.47	$p\sigma$, b
		1.59	$p\sigma$, b-d
	2.50	1.06	$p\sigma$, d
		1.23	$p\sigma$, d
		1.36	$p\sigma$, b
		1.39	$p\sigma$, b-d
		1.40	$p\sigma$, b
	3.00	1.33	$p\sigma$, d
		1.45	$p\sigma$, d
		1.54	$p\sigma$, d
		1.67	$p\sigma$, d
		1.77	$p\sigma$, b
		1.79	$p\sigma$, b
		2.33	$p\sigma$, d
		2.72	$p\sigma$, b

Table 4 continued

Source and test	Scale factor, λ	Mean stress ratio, σ^*_1/σ^*_2	Classification
Special ASTM Committee, Fourth Report [109], VNC (21)	1.67	1.05	$p\epsilon$, d
		1.09	$p\epsilon$, d
		1.10	$p\epsilon$, d
	3.00	0.98	$p\epsilon$, d
		1.13	$p\epsilon$, d
		1.37	$p\epsilon$, d
		1.42	$p\epsilon$, d
	3.67	1.05	$p\epsilon$, d
		1.24	$p\epsilon$, d
		1.52	$p\epsilon$, d
		1.78	$p\epsilon$, d
	5.00	1.08	$p\epsilon$, d
		1.30	$p\epsilon$, d
		1.65	$p\epsilon$, d
		2.00	$p\epsilon$, d
Special ASTM Committee, Fifth Report [110], CCP (7)	1.50	1.08	$p\sigma$, d
	2.00	1.16	$p\sigma$, d
	3.00	1.70	$p\sigma$, d
	4.00	1.93	$p\sigma$, d
Steigerwald [111], 3PB and 4PB (21)	1.04	0.88	$p\epsilon$, b-d
	1.14	0.94	$p\epsilon$, b-d
	1.17	0.88	$p\epsilon$, b-d
	1.23	0.86	$p\epsilon$, b-d
	1.27	0.91	$p\epsilon$, b-d
	1.30	0.98	$p\epsilon$, b-d
	1.35	0.99	$p\epsilon$, b-d
	1.72	1.16	$p\epsilon$, d
	2.10	1.36	$p\epsilon$, d
	2.26	1.44	$p\epsilon$, d
	2.47	1.96	$p\epsilon$, d
	2.84	1.57	$p\epsilon$, d
	3.24	1.65	$p\epsilon$, d
	3.62	1.75	$p\epsilon$, d
	3.90	1.65	$p\epsilon$, d
	4.12	1.74	$p\epsilon$, d
Sumpter [114], Bulge test (2)	6.00	1.12	$p\sigma$, d
Sunamoto *et al.*[115], CTS (21, T, a/w)	2.00	0.98	$p\epsilon$, d
		1.08	$p\epsilon$, d
		1.35	$p\epsilon$, b
		1.44	$p\epsilon$, d
		1.55	$p\epsilon$, d
		1.65	$p\epsilon$, d
	4.00	1.41	$p\epsilon$, d
Thomas *et al.* [116], CCP (8, T, a/w)	2.00	1.14	$p\sigma$, d
		1.18	$p\sigma$, d
		1.21	$p\sigma$, d
		1.33	$p\sigma$, d
Wei *et al.* [119], VNC (18, a/w)	1.20	0.99	$p\epsilon$, d
	1.25	0.98	$p\epsilon$, d
	1.27	1.09	$p\epsilon$, d
	1.48	1.10	$p\epsilon$, d
	2.00	1.12	$p\epsilon$, d
Weiss *et al.* [120], VNC (16)	1.84	1.21	$p\epsilon$, d
	4.21	2.30	$p\epsilon$, d
Weiss *et al.* [121], DEN and VNC (18, 1/56)	1.33	1.01	$p\epsilon$, d
	1.67	1.16	$p\epsilon$, d
	3.20	1.02	$p\epsilon$, d
	3.75	1.59	$p\epsilon$, d
	4.00	1.01	$p\epsilon$, d
		1.05	$p\sigma$, d
		1.09	$p\sigma$, d
		1.12	$p\sigma$, d
	5.00	1.85	$p\epsilon$, d
Wells [122], 3PB (8, 1/40)	1.42	1.33	$p\epsilon$, d
	2.01	1.00	$p\epsilon$, d
	2.84	0.97	$p\epsilon$, d
	4.00	1.10	$p\epsilon$, d
	5.68	1.10	$p\epsilon$, d
	8.03	1.16	$p\epsilon$, d
	11.3	1.16	$p\epsilon$, d

Table 4 continued

Source and text	Scale factor, λ	Mean stress ratio, σ_1^*/σ_2^*	Classification
Wessel [123],	2.00	1.03	pε, b
WOL (84,T)		1.11	pε, b
		1.12	pε, b
		1.20	pε, b-d
		1.23	pε, b
		1.30	pε, b
		1.30	pε, b
		1.35	pε, b
		1.36	pε, b
		1.40	pε, b
		1.41	pε, b
		1.41	pε, b-d
		1.48	pε, b
		1.50	pε, b-d
		1.52	pε, b
		1.56	pε, b
		1.59	pε, b
		1.62	pε, b-d
	3.00	2.06	pε, b
	4.00	1.75	pε, b
		1.83	pε, b
		1.85	pε, b
		2.14	pε, b
Wessel *et al.* [124],	1.20	1.02	pε, b
CTS (32, T)	1.33	1.35	pε, b
	1.50	1.27	pε, b
		1.51	pε, b
	1.67	1.11	pε, b
	2.00	1.26	pε, b
		1.58	pε, b
		1.71	pε, b
		1.76	pε, b
	2.50	1.02	pε, b
		1.62	pε, b
	4.00	0.99	pε, b-d
Wilson *et al.*	2.00	1.06	pσ, d
CCP (36, T)		1.09	pσ, d
		1.12	pσ, d
		1.13	pσ, d
		1.19	pσ, d
		1.27	pσ, d
	3.00	1.06	pσ, d
	4.00	1.03	pσ, d
		1.12	pσ, d
		1.20	pσ, d
		1.24	pσ, d
		1.28	pσ, d
		1.31	pσ, d
		1.33	pσ, d
		1.34	pσ, d
	6.00	1.12	pσ, d
		1.13	pσ, d
		1.22	pσ, d
		1.24	pσ, d
		1.28	pσ, d
		1.34	pσ, d
		1.35	pσ, d
		1.42	pσ, d
Winne and Wundt [127],	3.00	1.47	pσ, d
spinning disks (5)		1.96	pσ, b-d
Witt [128], CTS (10,T)	3.00	0.95	pε, d
		1.11	pε, d
	6.00	0.95	pε, d
		1.02	pε, d
	9.00	1.00	pε, d
		1.03	pε, d
Worthington [129],	2.11	1.15	pε, d
3PB (3,T)	4.72	1.80	pε, d

Table 4 continued

Source and test	Scale factor, λ	Mean stress ratio, σ_1^*/σ_2^*	Classification
Yukawa [130], 3PB (58, 1/15, T)	2.00	1.14	pε, d
	2.08	0.93	pε, d
	4.00	1.09	pε, b
		1.09	pε, b
		1.12	pε, d
		1.14	pε, b
		1.17	pε, b
		1.19	pε, b
		1.19	pε, b
		1.19	pε, d
		1.20	pε, b
		1.28	pε, b
		1.32	pε, d
		1.38	pε, b
		1.52	pε, b
		1.60	pε, b
		1.65	pε, b
		1.77	pε, b
	4.05	0.99	ε, d
	9.46	0.99	pε, d
	10.67	1.47	pε, b
		1.99	pε, b
		2.07	pε, b
		2.62	pε, b
		2.87	pε, b
		3.08	pε, b
Yukawa and McMullin [131], VNC (28, 1/16)	2.00	1.61	pε, d
	3.00	1.64	pε, d
		1.75	pε, d
	7.50	2.17	pε, d
		3.22	pε, d
Zhen-Yuan [134], TNF (3)	1.60	1.00	pε, d
	2.00	1.05	pε, d

Table 5. Sources and data for aluminum alloys

Source and test	Scale factor, λ	Mean stress ratio, σ^*_1/σ^*_2	Classification
Adams and Munro [6],	1.33	1.01	pσ, d
CCP and CTS (51)	1.49	0.95	pσ, d
	1.92	1.02	pσ, d
	1.98	0.99	pσ, d
	2.00	1.07	pσ, d
	2.47	1.01	pσ, d
	2.98	1.07	pσ, d
	3.85	1.08	pσ, d
	3.98	1.06	pσ, d
	5.77	1.17	pσ, d
Argy *et al.* [9],	1.39	1.04	pσ, d
CCP *and* CTS (8,T)	1.66	0.97	pσ, d
		1.18	pσ, d
Bonesteel [15],TNF (2)	1.60	1.12	pε, d
Bradshaw and Wheeler [17],	2.00	1.20	pσ, d
as reported in Newman [86],		1.24	pσ, d
CCP (8)	4.00	1.27	pσ, d
		1.48	pσ, d
	8.00	1.44	pσ, d
		1.96	pσ, d
Carman *et al.* [21],	5.00	1.12	pσ, d
CCP(30, 1/1000, a/w)		1.66	pσ, d
		1.78	pσ, d
		1.82	pσ, d
		1.96	pσ, d
		2.12	pσ, d
		2.29	pσ, d
		2.29	pσ, d
		2.49	pσ, d
Christian *et al.* [25],	2.00	1.31	pσ, d
CCP (6, 1/500, T)		1.47	pσ, d
	9.00	2.57	pσ, d
		2.85	pσ, d
Chu [26],	1.88	1.08	pε, d
4PB (29)		1.26	pε, d
		1.30	pε, d
	2.00	1.29	pε, b-d
		1.44	pε, b-d
	2.22	1.08	pε, d
		1.09	pε, d
	2.50	1.21	pε, d
		1.29	pε, d
		1.56	pε, b-d
		1.59	pε, b-d
	2.89	1.40	pε, d
	3.00	1.63	pε, b-d
	3.33	1.10	pε, d
	3.75	1.31	pε, d
		1.83	pε, b-d
	5.00	1.87	pε, d
	7.50	1.90	pε, d
Desisto *et al.* [30],	1.41	0.91	pε, d
VNC(9, 1/23)	1.59	1.29	pε, b-d
	2.23	1.03	pε, b
	2.24	0.94	pε, b-d
	3.16	1.02	pε, b
	4.46	1.05	pε, d
	10.00	1.31	pε, d
Eschweiler and Munz [32],	1.52	1.24	pε, d
Short bar (7)		1.25	pε, d
Forman [34]	1.20	1.00	pσ, d
as reported in Wang [99],	1.60	1.03	pσ, d
CCP (8)	2.40	1.06	pσ, d
Frediani [35],	1.52	1.03	pσ, d
CCP (7, a/w)		1.17	pσ, d
Heyer and McCabe [43],	2.00	1.11	pσ, b
		1.13	pσ, b
Hudson and Lewis [46], data supplied	1.33	1.19	pε, b
by Rockwell International, CTS (8)	1.67	1.37	pε, b

Table 5 continued

Source and test	Scale factor, λ	Mean stress ratio σ_1^*/σ_2^*	Classification
Jones *et al.* [49],	1.25	1.10	pε, b
VNC (141, 1/100, a/w)		1.11	pε, b-d
	1.50	1.03	pε, d
		1.05	pε, d
		1.06	pε, d
		1.16	pε, b-d
		1.21	pε, d
	2.00	1.03	pε, d
		1.06	pε, d
		1.08	pε, d
		1.08	pε, d
		1.10	pε, d
		1.18	pε, b-d
		1.31	pε, b-d
		1.31	pε, b-d
	3.00	1.09	pε, d
		1.13	pε, d
		1.14	pε, d
		1.38	pε, b-d
		1.55	pε, b-d
	4.00	1.11	pε, d
		1.18	pε, d
		1.20	pε, d
		1.64	pε, b-d
	5.00	1.15	pε, d
		1.25	pε, d
		1.27	pε, d
		1.84	pε, b-d
Kaufman [52],	2.00	1.32	pε, b
CTS (6)	3.00	1.50	pε, b
Kaufman [53],	2.13	1.17	pε, d
VNC (18, 1/147)		1.48	pε, b-d
Kaufman and Nelson [54],	1.50	1.11	pε, b
CTS (20)	2.00	1.16	pσ, b-d
		1.17	pε, b
		1.23	pε, b
		1.24	pσ, b-d
	3.00	1.42	pσ, b-d
Kaufman *et al.* [55],	2.13	1.25	pε, d
VNC (63, 1/100)		1.38	pε, d
		1.38	pε, d
Keller and Munz [56], CTS(2)	2.00	1.11	pε, b-d
Klier and Weiss [57],	1.67	1.00	pε, d
VNC (13, 1/44)	3.00	0.99	pε, d
	4.67	1.21	pε, d
	5.67	1.07	pε, d
		1.56	pε, d
Krafft *et al.*, CCP(2)	2.00	1.31	pσ, d
Lake [63], CTS(4)	4.00	1.79	pε, b
Lubahn [69], VNC(6, 1/160)	1.30	1.29	pε, d
Morozov [78],	1.67	1.34	pε, b-d
3PB (19)		1.36	pε, b
	1.85	1.36	pε, b
	2.00	1.21	pε, b-d
	2.07	1.34	pε, b
	2.22	1.28	pε, b
		1.55	pε, b-d
	2.52	1.43	pε, b-d
		1.44	pε, b
	3.03	1.46	pε, b-d
	4.00	1.67	pε, b-d
		1.81	pε, b
	4.11	1.76	pε, b-d
	6.56	1.80	pε, b-d
Munz [80],	2.00	1.08	pε, b-d
3PB (6)	4.00	1.24	pε, b-d
	8.00	1.50	pε, b-d
Nelson and Kaufman [84],	1.33	1.09	pε, d
3PB (32)		1.14	pε, d
		1.15	pε, d
		1.25	pε, d
		1.26	pε, d
		1.26	pε, d
		1.33	pε, d
		1.37	pε, d

Table 5 continued

Source and test	Scale factor, λ	Mean stress ratio, σ_1^*/σ_2^*	Classification
Nelson *et al.* [85],	1.33	1.10	pε, b
3PB and CTS (74)	1.50	1.07	pε, b-d
		1.07	pσ, b-d
		1.12	pε, b
		1.14	pε, b
		1.19	pε, b
		1.20	pσ, b-d
		1.21	pσ, b
		1.26	pσ, b
	2.00	0.88	pε, b-d
		1.26	pε, b
		1.28	pε, b
		1.37	pε, b-d
		1.43	pε, b
		1.56	pε, b
	3.00	0.94	pε, b-d
	4.00	0.76	pσ, d
		1.01	pε, b-d
		1.07	pε, b-d
Orange [87],	2.00	1.09	pσ, d
CCP (24, T, a/w)		1.13	pσ, d
		1.14	pσ, d
		1.15	pσ, d
		1.18	pσ, d
		1.19	pσ, d
		1.21	pσ, d
		1.32	pσ, d
	4.00	1.31	pσ, d
		1.39	pσ, d
		1.46	pσ, d
		1.52	pσ, d
		1.58	pσ, d
		1.63	pσ, d
		1.80	pσ, d
Poulose *et al.* [91],	1.50	1.07	pε, d
CTS (8)		1.12	pσ, d
	2.00	1.22	pε, d
		1.65	pσ, d
Poulose and Liebowitz [92],	2.00	1.03	pσ, d
CCP (8)	3.00	0.98	pσ, d
	6.00	1.08	pσ, d
	8.00	1.32	pσ, d
	10.00	1.32	pσ, d
	12.00	1.59	pσ, d
	14.00	1.75	pσ, d
Shannon *et al.* [104],	2.00	1.27	pε, b-d
DEN (22)		1.28	pε, b-d
		1.37	pσ, d
		1.89	pσ, d
Special ASTM Committee,	1.50	1.03	pε, d
Third Report [108],	2.00	1.03	pε, d
CCP and VNC (16)		1.06	pε, d
		1.07	pσ, d
		1.07	pσ, d
		1.20	pσ, d
		1.20	pσ, d
		1.24	pσ, d
		1.27	pσ, d
	2.50	1.13	pε, d
	3.00	1.30	pε, d
	3.20	1.38	pε, d
	4.00	1.29	pε, d
		1.46	pσ, d
		1.49	pε, d
	5.14	1.34	pσ, d
	8.00	1.87	pε, d
		2.04	pε, d
	13.71	1.85	pσ, d
Steigerwald [111],	1.43	1.18	pε, b-d
4PB and 3PB (16)	1.64	1.28	pε, b-d
	1.83	1.65	pε, b-d
	2.71	1.57	pε, b-d
	2.93	1.66	pε, b-d
	3.86	1.91	pε, b-d
	4.89	2.06	pε, b-d
		2.14	pε, b-d
Sullivan *et al.* [112], CTS (2)	1.67	1.07	pσ, d
Sullivan and Stoop [113],	1.67	1.18	pσ, d
CTS (13, a/w)		1.22	pσ, d

Table 5 continued

Source and test	Scale factor, λ	Mean stress ratio, $\dot{\sigma}_1/\dot{\sigma}_2$	Classification
Yusuff [132], from data in	1.50	1.19	pσ, b-d
Crichlow [29], McEvily		1.36	pσ, b-d
et al. [75] and Yusuff [133],	2.00	1.31	pσ, b-d
CCP (21)		1.36	pσ, d
		1.47	pσ, d
		1.47	pσ, d
	4.44	1.69	pσ, d
		2.01	pσ, d
		2.12	pσ, d
		2.37	pσ, d
	8.89	2.93	pσ, d
Wang [117],	1.33	1.09	pσ, d
CCP (8, a/w)	2.50	1.67	pσ, d
	3.33	1.82	pσ, d
Wang and McCabe [118],	1.33	1.12	pσ, d
CTS (5)	1.50	1.06	pσ, d
	3.33	1.85	pσ, d
Weiss *et al.* [120],	1.81	1.12	pϵ, d
VNC (33)	1.90	1.03	pϵ, d
	4.00	1.37	pϵ, d
		1.60	pϵ, d
Zinkham [135],	3.00	1.41	pσ, b
CCP (18)		1.50	pσ, b
Zinkham and Baughan [136],	1.67	1.14	pϵ, b
CTS (16)		1.18	pϵ, b

Table 6. Sources and data for other metals

Source and test	Scale factor, λ	Mean stress ratio, σ_1^*/σ_2^*	Classification
Chu [26],	1.33	0.98	pε, d
4PB and 3PB (56)	1.67	1.07	pε, d
		1.18	pε, d
	2.00	1.30	pε, d
	2.50	1.08	pε, d
		1.15	pε, d
		1.16	pε, d
		1.18	pε, d
		1.23	pε, d
		1.31	pε, d
		1.40	pε, d
	2.67	1.22	pε, d
		1.28	pε, d
		1.32	pε, d
		1.41	pε, d
	4.00	1.58	pε, d
	5.00	1.32	pε, d
		1.33	pε, d
		1.34	pε, d
		1.39	pε, d
		1.65	pε, d
		1.68	pε, d
	5.33	1.64	pε, d
	10.00	1.50	pε, d
		1.52	pε, d
		1.56	pε, d
		1.68	pε, d
		1.71	pε, d
		1.86	pε, d
		2.29	pε, d
DeSisto *et al.* [30],	1.41	1.03	pε, d
VNC (8 1/23)	2.23	1.08	pε, d
	2.24	1.19	pε, b-d
	3.15	1.45	pε, b-d
	3.16	1.19	pε, d
	5.00	1.76	pε, b-d
Gunderson [39], as reported in Newman [86], CTS (2)	1.33	1.03	pσ, b-d
Hall *et al.* [40], CTS (3)	2.67	1.55	pσ, b-d
Hilton [44],	2.50	1.38	pε, b
CTS (15)		1.40	pε, b-d
		1.48	pε, b-d
Klier and Weiss [57],	1.67	0.99	pε, d
VNC (10, 1/44)		1.05	pε, d
	3.00	0.95	pε, d
		1.09	pε, d
	4.67	1.00	pε, d
		2.31	pε, d
	5.67	1.02	pε, d
		2.40	pε, d
Munz *et al.* [81],	1.90	1.18	pε, b-d
3PB (102)		1.12	pσ, b
	2.00	1.29	pε, b-d
		1.31	pε, b
	3.17	1.73	pε, b-d
	3.84	1.54	pε, b
	4.20	1.60	pε, b
	7.80	2.09	pε, b
Payne [90],	3.00	1.86	pσ, d
CCP (8)		1.83	pσ, d
Repko *et al.* [94],	3.00	1.10	pσ, d
DEN (24)		1.46	pσ, d
		1.54	pσ, d
		1.63	pσ, b-d
Shannon *et al.* [104],	2.00	1.34	pσ, d
DEN (15)		1.44	pε, b-d
		1.48	pε, b-d
Special ASTM Committee,	2.31	1.43	pσ, b
Third Report [108],		1.52	pσ, b-d
CCP (30, 1/1000)	3.00	1.71	pσ, b-d
		1.81	pσ, d
Weiss *et al.* [121],	2.00	1.36	pσ, d
DEN (3, 1/45)	4.00	1.90	pσ, d

Table 7. Sources and data for nonmetals

Source and test	Scale factor, λ	Mean stress ratio, σ_1^*/σ_2^*	Classification
Bassim and Hsu [11], 3PB (5)	2.54	1.08	pε, b
Buresch [20], 4PB (40, 1/11)	2.33	1.32	pε, b
Costin [28],	2.00	1.40	pε, d
3PB (12)	4.00	1.78	pε, d
Kaplan [51],	2.00	1.83	pε, b
3PB and 4PB (27, a/w)		1.90	pε, b
		1.95	pε, b
		1.96	pε, b
		2.07	pε, b
		2.08	pε, b
		2.08	pε, b
		2.10	pε, b
		2.18	pε, b
Lewis and Smith [65], 4PB (6)	2.00	2.10	pε, b
McKinney and Rice [76],	1.17	1.47	pε, b
3PB (57, 1/12)	1.18	1.08	pε, b
	1.33	1.09	pε, b
		1.44	pε, b
	1.38	1.21	pε, b
	1.60	1.08	pε, b
		1.50	pε, b
	1.63	1.16	pε, b
	1.65	1.48	pε, b
	1.67	0.99	pε, b
	1.74	1.49	pε, b
	2.33	1.17	pε, b
	2.67	1.28	pε, b
	3.13	1.52	pε, b
	3.20	1.35	pε, b
	3.33	1.09	pε, b
	3.73	1.07	pε, b
	4.67	1.98	pε, b
	4.80	1.77	pε, b
Schmidt [99],	2.04	1.19	pε, d
3PB (16, a/w)		1.28	pε, d
	4.08	1.70	pε, d
Schmidt and Lutz [100],	2.00	1.28	pε, d
CTS and 3PB (22)		1.30	pε, d
	4.00	1.69	pε, d
		1.81	pε, d
	8.00	2.46	pε, d
Weiss *et al.* [120],	1.41	0.99	pε, b
4PB and VNC (46, 1/21)	1.44	1.17	pε, b
	1.80	1.49	pε, b
	2.18	1.43	pε, b
	2.69	1.37	pε, b
	2.84	1.75	pε, b
	3.42	1.61	pε, b
	3.98	2.08	pε, b
	5.95	1.70	pε, b
	6.26	2.63	pε, b
	11.31	2.25	pε, b
Williams and Ewing [125],	1.33	0.94	pσ, b-d
Pressure vessels (6)	1.50	1.84	pσ, b

SOME COMMENTS ON THE GRIFFITH-IRWIN APPROACH TO FRACTURE MECHANICS

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For brittle fracture, present day linear elastic fracture mechanics (LEFM) selects the stress intensity factor, K, as the parameter controlling damage. In essence this choice owes its origin to the classical thermodynamic or energy argument of Griffith, and the recognition of the equivalence of energy release rates and stress intensity factors by Irwin. While there now exist competing explanations for justifying K as the fracture controlling quantity, the underlying energy argument cannot be dismissed since the use of K to predict fracture implies the thermodynamic statement of Griffith, i.e. the connection is reversible. Accordingly an assessment of the validity of the energy balance approach is pertinent to an appraisal of LEFM even today.

A number of commentaries on the Griffith energy argument for brittle fracture are available in the literature and address various aspects of its consequences, e.g. Goodier in *Fracture*, Vol. II. The aspect of concern here is the size dependence implied by the approach. By virtue of having an energy source term which is one spatial dimension higher than the assumed sink, the balance always leads to a reduction in the predicted stress at fracture for scaled specimens as the inverse of the square root of some length. More precisely, for the uniaxial tension test Griffith's argument can be shown to result in

$$\sigma_u \propto \frac{1}{\sqrt{L}},$$

where σ_u is the ultimate stress, L is the length of the specimen: while for a cracked specimen with all its in-plane dimensions held in constant ratios to one another it gives

$$\sigma_* \propto \frac{1}{\sqrt{W}},$$

where σ_* is the applied stress at fracture, W is the width of the specimen. The question then arises as to how good such predictions are in practice. The answer is not too satisfactory. Specifically, the first, though representing a trend found to a limited extent in small specimens, is generally in complete disagreement with the physical evidence of size independence for the ultimate stress in sufficiently large specimens. And the second, on examination of over

300 experimental data for brittle and quasi brittle materials drawn from some forty odd references, is found to be complied with to within ±10% by less than 7% of the results. In all it would appear that the assumption of a surface-energy-like term as the sole energy sink in fracture proccesses in solids implicit in LEFM leads to an altogether too simple prediction of size effects - one that cannot really capture the variations in size dependence itself with size, or the sensitivity of size effects to different materials, or the way in which altering size by changing different dimensions enters into the effects. One explanation which has the potential of overcoming these shortcomings is to view size dependence as being governed by a highly stressed volume and admit Weibull-like dependencies. When these ideas are applied to various test geometries, including cracked test pieces, a consistent picture of strength size effects emerges.

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On Size Effects In Fracture

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Griffith's classical energy arguments [1], which form the basis of modern day fracture mechanics, imply a dependence of fracture stress on size. Specifically, for an in-plane scaled specimen pair (Fig.1), in theory we have

$$\sigma_1^*/\sigma_2^* = \sqrt{\lambda} \qquad (1)$$

where σ_1^*, σ_2^* are the applied stresses at fracture in Specimen 1, 2 and λ is the scale factor. That is, the larger the specimen the smaller the fracture stress. In contrast for out-of-plane dependence, the same *theory* gives

$$\sigma_t^*/\sigma_T^* = \sqrt{(1-\nu^2)}, \qquad (2)$$

where σ_t^*, σ_T^* correspond to plane stress, strain fracture stresses and ν is Poisson's ratio. The objective here is to examine how well these predictions are actually complied with.

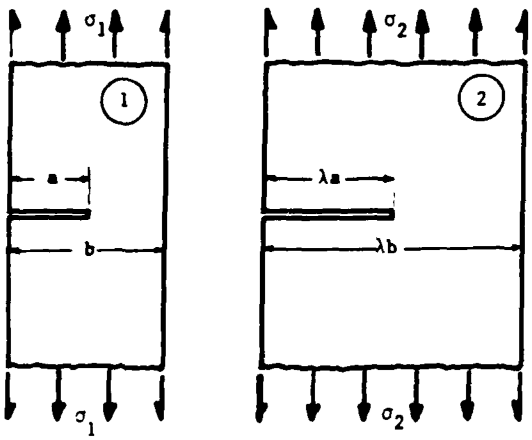

Fig.1. Scaled crack specimens

To this end Fig.2 presents in-plane size effects on the strength of cracked specimens from twenty references (see [2] for details). While some of the data lie close to equation (1) most do not, regardless of whether the data is for brittle or semi-brittle material, for plane stress or plane strain. Moreover scatter alone cannot account for the deviations from the theory (the bars in Fig.2 being 95% confidence limits for the associated data sets). Concerning thickness effects, there appears to be a lack of data for perfectly brittle solids to check if (2) holds. However, as is well known, nonbrittle data exhibit the opposite behavior to (2). While explanations of this trend exist none are generally recognized as being complete. In all, size predictions in fracture mechanics are in some conflict with the physical data.

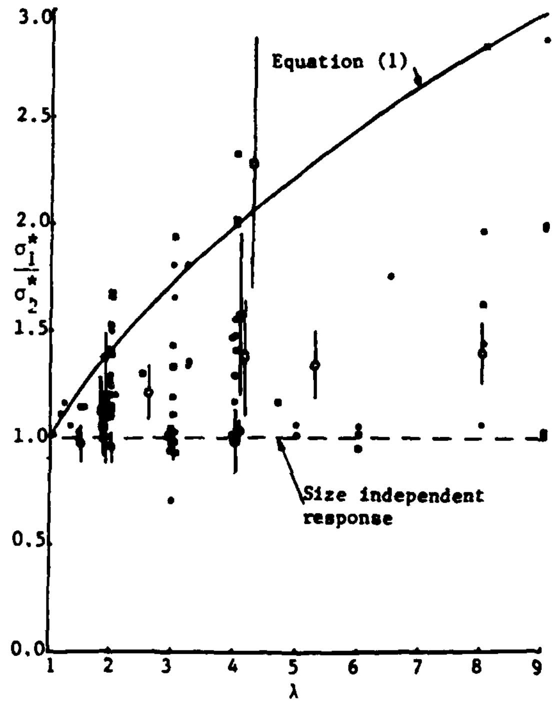

Fig.2. In-plane size dependence data

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By reviewing the basic arguments of fracture mechanics one can establish that the assumption of a *surface* term being the dominant energy sink in the thermodynamic condition for fracture is inappropriate. The question then arises is what is really happening in fracture size effects. One explanation follows on assuming cracks are not physically too different from other stress raisers. This enables their physical size dependence to be made compatible with size effects in general.

The general picture (Fig.3) of size effects has size independent and size dependent regimes. In the size independent regime fracture stress takes on the material handbook value, the ultimate stress σ_u. This value applies when V^*, the highly-stressed volume, is big enough. Consequently it is normally found via uniaxial tension tests wherein the entire volume can be highly stressed. If, though, this volume is sufficiently reduced ($V^*<V_u$), the size dependent regime is entered even using tensile tests (see e.g. Fig.3). In this regime models, such as those due to Weibull [3], provide reasonable data reduction schemes for predicting the increases in strength due to reductions in size.

Turning to other specimen types, if we make the nonunique but sensible definition that V^* be the volume seeing 95% or more of the maximum stress, we can obtain the estimates below for V^* in terms of the gross volume V:

$$V^* \sim V(\text{tension}),\ V^* \sim .05\ V\ (\text{bending}),$$
$$V^* \sim .05^2 V(\text{notches}),\ V^* \sim .05^n V(\text{cracks}). \quad (3)$$

Here n is an index whose range is approximately $2\leq n\leq 3$. As a result, bend, smooth notch, and crack specimens, in that order, can be expected

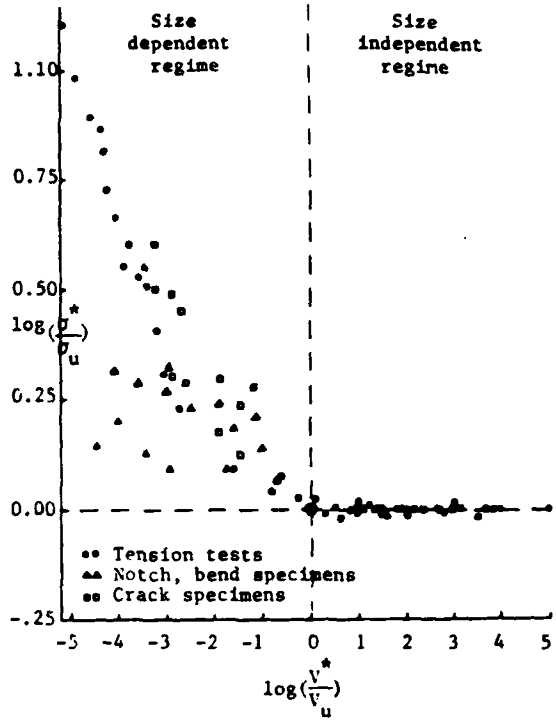

Fig.3. General fracture size effects

to more frequently fall in the size dependent regime; nonetheless given sufficiently large specimens all three eventually enter the size independent regime (see Figs.2,3). To date we have been able to get all data gathered to comply with this explanation. Further, this intepretation offers the potential of increased understanding of thickness effects.

In sum, size effects predicted in current fracture mechanics can disagree with the physical evidence and hence are capable of causing errors in practice, even nonconservative ones: the explanation put forward here would seem to be consistent with physical results and could thus become a part of an improved technology.

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